U.G. 1st Semester Examination-2019

MATHEMATICS

[HONOURS]

Course Code : MATH(H)/CC-1-T

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Symbols / Notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Find the eccentricity and the length of the latus rectum of the conic

$$\frac{2l}{r} = 5 - 2\cos\theta.$$

b) Find the point of inflection of the curve

$$y = \frac{x^3}{x^2 + a^2}.$$

c) If the origin is shifted to the point (1, –2) then find the transformed equation of $x^2 - y^2 = 5$.

d) Find the differential equation of all circles, which pass through the origin and whose centres are on the x-axis.

e) Verify whether the following differential equation is exact or not

$$(y^2 e^x + 2xy)dx - x^2 dy = 0.$$

[Turn over]

f) Find an integrating factor of the differential equation

$$\left(x^2y-2xy^2\right)dx+\left(3x^2y-x^3\right)dy=0.$$

g) Use L'Hospital rule to evaluate

$$\lim_{x\to0}\frac{\sin x-x}{x^3}.$$

h) Examine whether the origin is a Node or a Cusp of the curve

$$x^3+y^3=32xy.$$

i) Test whether the equation

$$4x^2-4xy+y^2-12x+6y+8=0$$

represent a pair of parallel straight lines.

j) Determine the length of an arc of the cycloid $x=a(\theta+\sin\theta),\ y=a(1-\cos\theta)$, measured from the vertex.

k) Find the asymptotes of the curve

$$r\cos\theta=2a\sin\theta.$$

l) Show that $y=x^4$ is concave upwards at the origin and $y=e^x$ is everywhere concave upwards.

m) Find the centre and radius of the sphere

$$2\left(x^2+y^2+z^2\right)-2x+4y-6z=15.$$

2. Answer any **four** questions: $5 \times 4 = 20$

a) A sphere of constant radius r passes through the origin and cuts the axes in A, B, C respectively. Prove that the centroid of the triangle ABC lies on the sphere

$$9\left(x^2 + y^2 + z^2\right) = 4r^2.$$

b) Find the equation to the generating lines of the paraboloid $(x+y+z)(2x+y-z) = 6z$, which pass through the point (1, 1, 1).

c) The circle $x^2 + y^2 = a^2$ revolves round the x-axis, show that the surface area and the volume of the whole sphere generated are respectively

$$4\pi a^2 \text{ and } \frac{4}{3}\pi a^3.$$

d) If any of the asymptotes of the curve

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, \left(h^2 > ab\right)$$

passes through the origin, prove that

$$af^2 + bg^2 = 2fgh.$$

e) If the centre of a circle lies upon the parabola $y^2 = 4ax$ and the circle passes through the vertex of the parabola, show that the envelope of the circle is $y^2(2a + x) + x^3 = 0$.

f) Solve : $y^2 + \left(x - \frac{1}{y}\right)\frac{dy}{dx} = 0$.

3. Answer any **two** questions: 10×2=20

a) i) If the straight line $r\cos(\theta-\alpha)=p$ touches the parabola $\frac{l}{r}=1+\cos\theta$, show that $p=\frac{l}{2}\sec\alpha$. 5

ii) Find the area bounded by the curves $y^2-4x-4=0$ and $y^2+4x-4=0$. 5

b) i) Solve : $xy-\frac{dy}{dx}=y^3e^{-x^2}$. 5

ii) Find the condition that the plane $lx+my+nz=p$ is a tangent plane to the paraboloid $ax^2+by^2=2z$. 5

c) i) If $I_n=\int_0^{\pi/2}\cos^n x\,dx$ where n is a positive integer, show that $I_n=\frac{n-1}{n}I_{n-2}$.

Hence find the value of

$$\int_0^{\pi/2}\cos^4 x\sin^2 x\,dx.$$ 3+2

ii) Determine the value of a, b, c so that $\frac{(a+b\cos x)x-c\sin x}{x^5}\to 1$ as $x\to 0$. 5
