U.G. 1st Semester Examination-2018 MATHEMATICS (HONOURS)

Course Code: MTMH/CC-T-I

Full Marks: 60

Time: $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

1. Answer any ten questions:

 $2 \times 10 = 20$

- a) If the origin is shifted to the point (0, 1) then find the transformed equation of $y^2-y=0$.
- b) For what value of λ , does the equation $xy+5x+\lambda y+15=0$ represents a pair of straight lines?
- c) Test whether

$$(\cos y + y \cos x) dx + (\sin x - x \sin y) dy = 0$$

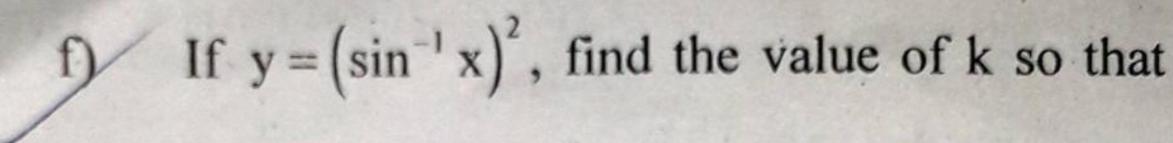
is exact or not.

d) Find the differential equation of the system of circles having a constant radius and whose centres lie on x-axis.

Find the asymptotes of the curve

$$xy^2-yx^2 = x+y+1.$$

[Turn over]



$$(1-x^{2})\frac{d^{2}y}{dx^{2}}-x\frac{dy}{dx}+k=0.$$

Show that sec x is an integrating factor of the equation

$$\cos x \frac{dy}{dx} + y \sin x = 1.$$

h) Find the nature of the conic

$$r = \frac{1}{4 - 5\cos\theta}.$$

i) Find the points of inflection of the curve

$$y = \frac{x^3}{a^2 + x^2}$$

j) Evaluate
$$\lim_{x\to 1} \left(\frac{1}{x^2-1} - \frac{2}{x^4-1} \right)$$
.

- k) Show that $y = x^4$ is concave upwards at the origin and $y = e^x$ is everywhere concave upwards.
- 1) Define singular point and characteristic point of a curve $f(x, y, \alpha) = 0$ where α is fixed.
- m) What will be the form of the equation $x^2 y^2 = 4$, if the coordinate axes are rotated through an angle $-\frac{\pi}{2}$?

n) Evaluate
$$\int_{0}^{4} \int_{0}^{1} xy(x-y) dy dx$$
.

- Obtain the singular solution of the differential equation $y = px + \frac{a}{p}$, where $p = \frac{dy}{dx}$.
- 2. Answer any four questions:

$$5 \times 4 = 20$$

a) Find the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
.

Find the asymptotes of the curve $x^2y-xy^2+xy+y^2+x-y=0$,

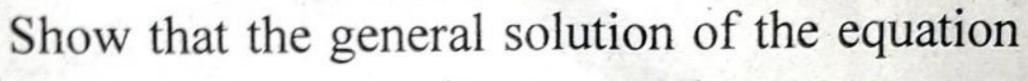
and show that they cut the curve again in three points which lie on the curve x+y=0.

- Show that the envelope of circles whose centres lie on the rectangular hyperbola $xy = c^2$ and which pass through its centre is $(x^2 + y^2)^2 = 16c^2xy$.
- Find the equation of the sphere which passes through the point (1, -2, 3) and through the circle $x^2+y^2+z^2=9$, 2x-3y+5z=8. Find its centre and radius.

Prove that the section of the conicoid $ax^2 + by^2 + cz^2 = 1$ by a tangent plane to the cone

$$\frac{x^{2}}{b+c} + \frac{y^{2}}{c+a} + \frac{z^{2}}{a+b} = 0$$

is a rectangular hyperbola.



$$\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{Py} = \mathrm{Q}$$

can be written in the form y=k(u-v)+v, where k is a constant and u and v are its two particular solutions.

3. Answer any two questions:

$$10\times2=20$$

a) i) Prove that $\frac{1}{(x+y+1)^4}$ is an integrating

factor of

$$(2xy-y^2-y)dx+(2xy-x^2-x)dy = 0$$

and hence solve it.

ii) Prove that the area of the section of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, by the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \text{ is } \frac{2\pi}{3\sqrt{3}} \sqrt{\left(b^2c^2 + c^2a^2 + a^2b^2\right)}.$$

b) i) A chord LM of a conic with eccentricity e and semi-latus rectum *l* subtends a right angle at a focus S. Show that

$$\left(\frac{1}{\mathrm{SL}} - \frac{1}{l}\right)^2 + \left(\frac{1}{\mathrm{SM}} - \frac{1}{l}\right)^2 = \frac{\mathrm{e}^2}{l^2}.$$

ii) If
$$I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$$
, show that
$$I_{n+1} + I_{n-1} = \frac{1}{n}$$
. Hence evaluate
$$\int_0^{\frac{\pi}{4}} \tan^8 x \, dx$$
. 5+5

- c) i) Evaluate $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$ by L'Hospitals's rule.
 - Find the volume of the solid generated by revolving the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 + \cos \theta)$ about x-axis. 5+5
 - d) i) If $y = x^{n-1} \log x$, then show that

$$y_n = \frac{(n-1)!}{x}$$
, where $y_n = \frac{d^n y}{dx^n}$.

ii) Find the length of the curve $y = \frac{a}{2} \left(e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right) \text{ from } x = 0 \text{ to } x = x.$