U.G. 1st Semester Examination-2018

MATHEMATICS

(HONOURS)

Course Code : MTMH/CC-T-I

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

1. Answer any **ten** questions: $2\times10=20$

a) If the origin is shifted to the point (0, 1) then find the transformed equation of $y^2-y=0$.

b) For what value of λ, does the equation $xy+5x+\lambda y+15=0$ represents a pair of straight lines?

c) Test whether

$$(\cos y+y\cos x)dx+(\sin x-x\sin y)dy=0$$

is exact or not.

d) Find the differential equation of the system of circles having a constant radius and whose centres lie on x-axis.

e) Find the asymptotes of the curve

$$xy^2-yx^2=x+y+1.$$

[Turn over]

f) If $y = (\sin^{-1} x)^2$, find the value of k so that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k = 0.$$

g) Show that sec x is an integrating factor of the equation

$$\cos x\frac{dy}{dx} + y\sin x = 1.$$

h) Find the nature of the conic

$$r = \frac{1}{4 - 5\cos\theta}.$$

i) Find the points of inflection of the curve

$$y = \frac{x^3}{a^2 + x^2}.$$

j) Evaluate $\lim_{x \to 1}\left(\frac{1}{x^2 - 1} - \frac{2}{x^4 - 1}\right)$.

k) Show that $y = x^4$ is concave upwards at the origin and $y = e^x$ is everywhere concave upwards.

l) Define singular point and characteristic point of a curve $f(x, y, \alpha) = 0$ where α is fixed.

m) What will be the form of the equation $x^2 - y^2 = 4$, if the coordinate axes are rotated through an angle $-\frac{\pi}{2}$?

n) Evaluate $\int_0^4 \int_0^1 xy(x-y)\,dy\,dx$.

o) Obtain the singular solution of the differential equation $y = px + \frac{a}{p}$, where $p = \frac{dy}{dx}$.

2. Answer any **four** questions: 5×4=20

a) Find the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

b) Find the asymptotes of the curve

$$x^2y - xy^2 + xy + y^2 + x - y = 0,$$

and show that they cut the curve again in three points which lie on the curve $x+y = 0$.

c) Show that the envelope of circles whose centres lie on the rectangular hyperbola $xy = c^2$ and which pass through its centre is $(x^2 + y^2)^2 = 16c^2xy$.

d) Find the equation of the sphere which passes through the point (1, –2, 3) and through the circle $x^2+y^2+z^2 = 9$, $2x-3y+5z = 8$. Find its centre and radius.

e) Prove that the section of the conicoid $ax^2 + by^2 + cz^2 = 1$ by a tangent plane to the cone

$$\frac{x^2}{b+c} + \frac{y^2}{c+a} + \frac{z^2}{a+b} = 0$$

is a rectangular hyperbola.

f) Show that the general solution of the equation

$$\frac{dy}{dx} + Py = Q$$

can be written in the form $y=k(u-v)+v$, where k is a constant and u and v are its two particular solutions.

3. Answer any **two** questions: $10\times2=20$

a) i) Prove that $\frac{1}{(x+y+1)^4}$ is an integrating factor of

$$(2xy-y^2-y)dx+(2xy-x^2-x)dy = 0$$

and hence solve it.

ii) Prove that the area of the section of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ is $\frac{2\pi}{3\sqrt{3}}\sqrt{(b^2c^2+c^2a^2+a^2b^2)}$.

5+5

b) i) A chord LM of a conic with eccentricity e and semi-latus rectum l subtends a right angle at a focus S. Show that

$$\left(\frac{1}{SL}-\frac{1}{l}\right)^2+\left(\frac{1}{SM}-\frac{1}{l}\right)^2=\frac{e^2}{l^2}.$$

ii) If $I_n=\int_0^{\frac{\pi}{4}}\tan^n x\,dx$, show that $I_{n+1}+I_{n-1}=\frac{1}{n}$. Hence evaluate $\int_0^{\frac{\pi}{4}}\tan^8 x\,dx$. 5+5

c) i) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$ by L'Hospitals's rule.

ii) Find the volume of the solid generated by revolving the cycloid $x=a(\theta+\sin\theta)$, $y=a(1+\cos\theta)$ about x-axis. 5+5

d) i) If $y=x^{n-1}\log x$, then show that

$$y_n=\frac{(n-1)!}{x}, \text{ where } y_n=\frac{d^n y}{dx^n}.$$

ii) Find the length of the curve

$$y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$$ from x = 0 to x = x.

5+5