U.G. 1st Semester Examination-2018

MATHEMATICS

(HONOURS)

Course Code : MTMH/CC-T-II

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Symbols have their usual meanings.

1. Answer any **ten** questions: 2×10=20

i) Show that gcd(a, a+2) =1 or 2 for every integer a.

ii) Find the modulus and amplitude of $\sin\theta + i\cos\theta$.

iii) Prove that

$$\frac{(\cos 2\alpha + i\sin 2\alpha)^4 (\cos 3\alpha - i\sin 3\alpha)^2}{(\cos 4\alpha + i\sin 4\alpha)^3 (\cos 5\alpha + i\sin 5\alpha)^2} = 1.$$

iv) If α, β, γ are the roots of the equation $x^3 + px + q = 0$, then find the value of $\sum \frac{1}{\alpha+\beta}$.

v) Find the greatest value of x+y+z subject to the condition $x^3+y^3+z^3 = 6$, where x, y, z are positive real numbers.

[Turn over]

vi) Let f, g: $\mathbb{R} \to \mathbb{R}$ be two mappings defined by $f(x) = |x| + x,\ x \in \mathbb{R}$ and $g(x) = |x| - x,\ x \in \mathbb{R}$. Find $f \circ g$.

vii) Solve the equation $6x^3 - 11x^2 + 6x - 1 = 0$, considering the roots are in A.P.

viii) Find the remainder when 7^{30} is divided by 4.

ix) If ω is an imaginary cube root of unity then find the value of $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$.

x) Check whether the mapping $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x, y) = (x+y, y, x)$ for all $(x, y) \in \mathbb{R}^2$, is a linear transformation or not.

xi) State fundamental theorem of arithmetic.

xii) Find the matrices A and B, where

$$2A+3B=\begin{pmatrix} 8 & 3 \\ 7 & 6 \end{pmatrix} \text{ and } A+B^t=\begin{pmatrix} 3 & 1 \\ 3 & 3 \end{pmatrix}.$$

xiii) Find the condition for which the two roots of the equation $x^3 + px^2 + qx + r = 0$ are equal in magnitude but opposite in sign.

xiv) Determine k so that the vectors (1, −1, 2), (0, k, 3) and (−1, 2, 3) are linearly independent.

xv) If a mapping $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = -x^2 + 5,\ x \in \mathbb{R}$, then find $f^{-1}(6)$.

2. Answer any **four** questions: 5×4=20

i) Solve the equation

$$x^3 - 3x^2 + 12x + 16 = 0.$$

ii) Show that the points corresponding to the roots of the equation $Z^n = (1+Z)^n$ are collinear.

iii) Let $A = \{x \in \mathbb{R} : -1 < x < 1\}$ and define a mapping $f : \mathbb{R} \to A$ by $f(x) = \frac{x}{1+|x|}$, $x \in \mathbb{R}$. Show that f is bijective and find f^{-1}.

iv) State Cayley Hamilton Theorem. Using that theorem find A^{50}, where $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

v) Find the values of a and b for which the following system of equations:

$$x+4y+2z = 1$$
$$2x+7y+5z = 2b$$
$$4x+ay+10z = 2b-1$$

has

a) a unique solution,

b) no solution,

c) infinite number of solutions

over the field of rational numbers.

vi) Define equivalence relation on a set. If R is an equivalence relation on a set A, then prove that R^{-1} is also an equivalence relation on A.

Answer any **two** questions from **Question No.3** to **Question No. 6:** $10 \times 2 = 20$

3. a) Find the rank of the matrix by reducing to its echelon form $\begin{bmatrix} 1 & -1 & -1 & 0 \\ -1 & 0 & -4 & 1 \\ -1 & 1 & 1 & -2 \\ 2 & -1 & 3 & 1 \end{bmatrix}$.

b) If A is a real orthogonal matrix of order n and $I_n + A$ is non-singular, then prove that $(I_n + A)^{-1}(I_n - A)$ is skew-symmetric.

c) Solve the equation

$$x^4 - 4x^3 + 16x^2 + 48x - 336 = 0.$$

3+3+4

4. a) Apply Descarte's rule of signs to find the nature of roots of the equation

$$x^4 + 16x^2 + 7x - 11 = 0.$$

b) Find the matrix A if

$$\text{adj } A = \begin{pmatrix} 1 & 3 & -4 \\ -2 & 2 & -2 \\ 1 & -3 & 4 \end{pmatrix}.$$

c) Show that n^3-n is divisible by 6 for all $n \in \mathbb{N}$.

3+4+3

5. a) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T(x, y, z) = (3x-2y, -2x+3y, 5z),$$

for all $(x, y, z) \in \mathbb{R}^3$. Determine whether T is a linear transformation or not. If linear, find it's matrix T relative to the basis

$$B=\{(1, 1, 0), (1, 0, 1), (0, 1, 1) \text{ of } \mathbb{R}^3.$$

b) In $\mathbb{Z}$ define $x \rho y$ if and only if x^2-y^2 is a multiple of 5. Show that ρ is an equivalence relation. Hence find the corresponding partition of $\mathbb{Z}$.

c) If $z_1^2 + z_2^2 + z_3^2 - z_1z_2 - z_2z_3 - z_3z_1 = 0$, then prove that

$$|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1|$$

where z_1, z_2, z_3 are three complex numbers.

(1+2)+4+3

6. a) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ then form the equation whose roots are $\beta^2 + \beta\gamma + \gamma^2$, $\gamma^2 + \gamma\alpha + \alpha^2$, $\alpha^2 + \alpha\beta + \beta^2$.

b) Find all the eigenvalues and eigenvectors of the following matrix

$$\begin{pmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & 4 \end{pmatrix}.$$

c) For a positive integer m, prove that $ax \equiv ay \pmod{m}$ if and only if $x \equiv y\left(\bmod \frac{m}{d}\right)$, where a, x, y are integers and $d = \gcd(a, m)$.

3+4+3