U.G. 5th Semester Examination - 2022

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-1A

(Linear Programming)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

1. Answer any **ten** questions: $2\times10=20$

   i) Find the basic feasible solutions of the equations

   $$x_1 + 2x_3 = 1$$

   $$x_2 + x_3 = 4.$$

   ii) Write down the relationship between the optimum value of maximization and minimization type problems.

   iii) Express the following LPP as standard maximization problem:

[Turn Over]

Minimize $Z = 4x_1 + 3x_2 + 2x_3$

subject to $x_1 + 4x_2 - x_3 \leq 7$

$x_1 - 3x_2 + 2x_3 \leq 12$

$2x_1 + x_2 + x_3 = 8$

$4x_1 + 7x_2 - x_3 \geq 16$

$x_1, x_2, x_3 \geq 0$.

iv) What is the relation between the optimal values of primal and dual problems (assume that both exist)?

v) Write down the dual of the following problem:

Maximize $Z = 2x_1 + 3x_2 + x_3$

subject to $2x_1 + x_2 - 3x_3 \leq 7$

$2x_1 - x_2 + x_3 \leq 6$

$x_1 + 3x_2 + x_3 \leq 8$

$2x_1 + 3x_2 - x_3 \geq 12$

$x_1, x_2, x_3 \geq 0$,

x_2 is unrestricted in sign.

vi) Show that a hyperplane is a convex set.

vii) Extreme points are finite in number—Justify.

viii) Prove that the set defined by $X = \{x : |x| \leq 7\}$ is a convex set.

ix) Find out the extreme points (if any) of the convex set $S = \{(x, y : |x| \le 2, |y| \le 1)\}$.

x) Is the point (1,10) in the convex set of feasible solutions determined by the constraints

$2x_1 + 5x_2 \le 40$, $x_1 + x_2 \le 11$, $x_2 \ge 4$, $x_1, x_2 \ge 0$,

a solution?

xi) State fundamental theorem of LPP.

xii) What is the criterion for the existence of multiple optimal solutions for Transportation Problem?

xiii) What is unbalanced Transportation Problem? How can you convert it into a balanced Transportation Problem?

xiv) Define a symmetric game. Why is it called so?

xv) State Fundamental theorem of Game Theory.

2. Answer any **four** questions: 5×4=20

i) Solve the following L.P.P. by simplex method:

Maximize $Z = 60x_1 + 50x_2$

subject to $x_1 + 2x_2 \le 40$

$3x_1 + 2x_2 \le 60$

$x_1, x_2 \ge 0$.

ii) Solve the following LPP by graphical method:

Maximize $Z = 10x_1 + 15x_2$

subject to $x_1 + x_2 \geq 2$

$3x_1 + 2x_2 \leq 6$

$x_1, x_2 \geq 0$.

iii) Show that the feasible solution $x_1 = 1$, $x_2 = 1$, $x_3 = 0$ and $x_4 = 2$ to the system

$$x_1 + x_2 + x_3 = 2$$

$$x_1 + x_2 - 3x_3 = 2$$

$$2x_1 + 4x_2 + 3x_3 - x_4 = 4$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

is not basic.

iv) For what value of a, the game with the following payoff matrix is strictly determinable?

A

B		I	II	III
	I	a	5	2
	II	-1	a	-8
	III	-2	3	a

v) State and prove weak duality theorem.

vi) Find out the optimal (minimum) assignment cost from the following cost matrix:

	I	II	III	IV
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

3. Answer any **two** questions: 10×2=20

i) a) Solve the following transportation problem:

	D_1	D_2	D_3	
O_1	50	30	220	1
O_2	90	45	170	3
O_3	270	200	50	4
	4	2	2	

b) Show that

$$S=\{(x_1, x_2, x_3): 2x_1 - x_2 + x_3 \leq 4,\ x_1 + 2x_2 - x_3 \leq 1\}$$

is a convex set. 6+4=10

ii) Solve the following L.P.P. by using Big-M method: 10

$$\text{Maximize } Z = 5x_1 - 2x_2 + 3x_3$$

$$\text{subject to } 2x_1 + 2x_2 - x_3 \geq 2$$

$$3x_1 - 4x_2 \leq 3$$

$$x_2 + 3x_3 \leq 5$$

$$x_1,\ x_2,\ x_3 \geq 0.$$

iii) Write down the dual of the following problem: 10

$$\text{Minimize } Z = 3x_1 + 4x_2$$

$$\text{subject to } x_1 + x_2 \leq 12$$

$$2x_1 + 3x_2 \leq 21$$

$$x_1 \leq 8$$

$$x_2 \leq 6$$

$$x_1,\ x_2 \geq 0.$$

Solving the dual problem, discuss the nature of solution of the primal problem.

iv) a) Prove that, the Transportation Problem always has a feasible solution.

b) Reduce the following game to 2×2 game and then solve it:

$$\begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix}$$
