U.G. 5th Semester Examination - 2022

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-2A

(Probability and Statistics)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Prove that the correlation coefficient ρ lies between [−1, 1].

b) Can the function

$$F(Y)=\begin{cases} k\left(1-y^2\right) & 0\le y\le 1 \\ 0 & otherwise \end{cases}$$

be a distribution function? Explain.

c) Find the characteristic function of a random variable Y having density function $g(x) = Be^{-\theta|x|}$, $-\infty < y < \infty$, where $\theta > 0$ and B is a suitable constant.

d) State Chebyshev's inequality for a continuous random variable.

e) Find the expectation of a discrete random variable Z whose probability function is given by $h(z) = (1/4)^z$, $(z = 1, 2, 3, \ldots)$.

f) A random variable X has the density function

$$f(x) = \frac{r}{1+x^2}, \text{ where } -\infty < x < \infty.$$

i) Find the value of the constant r

ii) Find the probability that X^2 lies between 1/3 and 1

g) Define conditional expectation.

h) Find the probability of drawing three kings at random from a deck of 52 ordinary cards if the cards are (i) not replaced and (ii) replaced.

i) Let the random variable X has density function

$$f(x) = \begin{cases} 1/(b-c) & b \le x \le c \\ 0 & otherwise \end{cases}.$$ Find the k^{th} moment about (i) the mean, (ii) the origin.

j) Find the probability of not getting a 7 or 9 total on either of two tosses of a pair of fair dice.

k) Define coefficient of skewness and kurtosis of a distribution.

l) Find (i) the covariance, (ii) correlation coefficient of two random variables X_1 and X_2 if $E(X_1)=2$, $E(X_2)=3$, $E(X_1X_2)=10$, $E(X_1^2)=9$, $E(X_2^2)=16$.

m) The joint density function of two continuous random variables X_1 and X_2 is

$$f(x_1, x_2) = \begin{cases} 2\beta x_1 x_2 & 0 < x_1 < 2,\ 1 < x_2 < 9 \\ 0 & \textit{otherwise.} \end{cases}$$

Find the value of the constant β.

n) In tossing a pair of fair dice, find the expectation of the sum of points.

o) If $Y^* = (Y - \mu)/\sigma$ is a standardized random variable, prove that

i) $E(Y^*) = 0$,

ii) $Var(Y^*) = 1$.

2. Answer any **four** questions: $5 \times 4 = 20$

a) State and prove law of large numbers theorem.

b) Find the probability of getting a total of 10 (i) once, (ii) twice, in two tosses of a pair of fair dice.

c) The random variable X can assume the values 1 and -1 with probability $\frac{1}{2}$ each. Find

i) the moment generating function,

ii) the first four moments about the origin.

d) If X_1 and X_2 are independent random variables, then show that

$$E(X_1 X_2) = E(X_1)E(X_2).$$

e) Define type-I and type-II errors.

The probability density function of the random variable Y is

$$f(y) = \begin{cases} \frac{1}{\theta} e^{-y/\theta}, & y > 0 \\ 0, & y \le 0, \end{cases}$$

where $\theta > 0$. For testing the hypothesis $H_0 : \theta = 3$ against $H_A : \theta = 5$, a test is given as "Reject H_0 if $Y \ge 4.5$". Find the probability of type-I error and power of the test.

f) Show that if a binomial distribution with n=100 is symmetric, its coefficient of kurtosis is 2.9.

3. Answer any **two** questions: $10\times2=20$

a) Let X_1 and X_2 be independent random variables having density function

$$f(\eta)=\begin{cases}2e^{-2\eta} & \eta\geq0\\ 0 & otherwise.\end{cases}$$

Find $E(X_1+X_2)$, $E(X_1^2+X_2^2)$ and $E(X_1X_2)$.

Does (i) $E(X_1+X_2)=E(X_1)+E(X_2)$, (ii) $E(X_1X_2)=E(X_1)E(X_2)$? Explain. 5+5

b) Find the probability that in 220 tosses of a fair coin (i) between 45% and 55% will be heads, (ii) or more will be heads. 5+5

c) If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals, (i) exactly 3, (ii) more than 2, individuals will suffer a bad reaction.

d) A continuous random variable X has probability density function given by

$$f(u)=\begin{cases}2e^{-2u} & u\geq 0\\ 0 & otherwise.\end{cases}$$

Find (a) $E(X)$, (b) $E(X^2)$, (c) the variance and (d) the standard deviation for the random variable X. 5+5
