U.G. 2nd Semester Examination - 2019

MATHEMATICS

[GENERIC ELECTIVE]

Course Code : MTMH/GE-T-02

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: $2\times10=20$

a) Determine the order, degree, linearity and the independent variable of the equation:

$$\frac{d^3y}{dx^3}-15x\frac{dy}{dx}=e^x+2.$$

b) Eliminate the parameters a and b from the following primitives:

$$xy=ae^x+be^{-x}.$$

c) Find the primitive of the equation $\frac{dy}{dx}=-\frac{y}{x}$. Find a particular solution that will satisfy the initial condition y = 1, when x = 1.

d) Solve: $(x^2-y)dx+(y^2-x)dy=0$.

[Turn Over]

e) State the necessary and sufficient condition for the equation

$$Mdx + Ndy = 0,$$

where M and N are functions of x and y, to be exact. Is the equation $(x^2 - y)dx + (x - y^2)dy = 0$ exact?

f) Solve: $(x^2 + 1)(y^2 - 1)dx + xydy = 0$.

g) Solve: $x^2p^2 - 2xyp + y^2 = x^2y^2 + x^4$.

Here $p = \frac{dy}{dx}$.

h) Solve: $(D^2 - 1)y = e^{2x}$.

i) Solve: $\frac{d^2y}{dx^2} = -9y$.

j) Solve: $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.

k) Eliminating the arbitrary constants a and b from the equation $z = (a+x)(b+y)$ from the p.d.e. Determine its order and degree.

l) Form the partial differential equation by eliminating the arbitrary function F from the equation $z = F(x^2+y^2)$.

m) Solve: $p + q = x$.

n) Classify the partial differential equation

$$2xy\frac{\partial^2 z}{\partial x\partial y}-\left(x^2+y\right)\frac{\partial^2 z}{\partial y^2}-\left(x+y\right)z=0$$

at the point (–1, 1).

o) Solve: pd = 10.

2. Answer any **four** questions: 5×4=20

a) Solve: $\frac{dy}{dx}-y^2=x^2-2xy.$

b) Solve: $\frac{dy}{dx}=\frac{y-x+1}{y+x+5}.$

c) Solve: $\frac{d^2y}{dx^2}+9y=\text{Sec}3x.$

d) Solve the following system of simultaneous equations:

$$Dx-7x+y=0$$

$$Dy-2x-5y=0.$$

Where $D\equiv\frac{d}{dt}$

e) Find the general solution of the partial differential equation $(y+zx)\,p-(x+yz)q=x^2-y^2$.

f) Find the particular solution of the equation $\left(y-z\right)\frac{\partial z}{\partial x}+\left(z-x\right)\frac{\partial z}{\partial y}=x-y$ which passes through the curve $xy=4$, $z=0$.

3. Answer any **two** questions: 10×2=20

a) i) Solve: $(y-x)(1+x^2)^{1/2}\frac{dy}{dx}=4(1+y^2)^{3/2}$.

ii) Solve: $(y^4-2x^3y)dx+(x^4-2xy^3)dy=0$.

5+5

b) i) Solve: $D^2(D^2+D+1)y=x^2$.

ii) Solve: $(D^2-2D)y=e^x\cos x$ by the method of variation of parameters. 4+6

c) i) Solve: $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$.

ii) Solve: $(x+a)^2\frac{d^2y}{dx^2}-4(x+a)\frac{dy}{dx}+6y=x$.

Here a = constant. 5+5

d) i) Solve the p.d.e $xp+3yq=2(z-x^2q^2)$ by Charpits method.

ii) Determine the region in R^2 where the partial differential equation

$$(x^2-1)\frac{\partial^2 z}{\partial x^2}+2y\frac{\partial^2 z}{\partial x\partial y}-\frac{\partial^2 z}{\partial y^2}=0$$

is hyperbolic, elliptic, parabolic. 7+3