U.G. 1st Semester Examination - 2022

MATHEMATICS

[PROGRAMME]

Course Code : MATH-G-CC-T-01

(Algebra and Analytical Geometry)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

GROUP–A

[Marks : 20]

1. Answer any **ten** questions: 2×10=20

a) Find cube root of 1.

b) z is a complex number satisfying the condition $\left|z-\frac{3}{z}\right|=2$. Find the greatest value of $|z|$.

c) Find the principal value of $(1+i)^i$.

d) A relation ρ is defined on the set $\mathbb{Z}$ by "$a\rho b$ if and only if $a-b$ is divisible by 5" for $a, b \in \mathbb{Z}$. Show that ρ is an equivalence relation.

e) Determine whether the permutation f on the set $\{1,2,3,4,5,6\}$ is odd or even, where $f=\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 2 & 3 & 5 & 6 & 1 & 4\end{pmatrix}$.

f) If b be an element of a group (G, o) and $o(b)=20$, find the order of the element b^6.

g) Why $(Z, .)$ is not a group?

h) Solve the equation $2x^3-x^2-18x+9=0$, if two roots are equal in magnitude but opposite in sign.

i) Apply Descartes' rule of signs to find the nature of the roots of the equation $x^4+2x^2+3x-1=0$.

j) Find the angle of rotation through which the axes must be turned so that the equation $lx-my+n=0$, $(m \neq 0)$ may be reduced to form $ay+b=0$.

k) The gradient of one of the straight lines of $ax^2+2hxy+by^2=0$ is twice of the other. Show that $8h^2=9ab$.

l) Find the center and diameter of the circle

$$r=3sin\theta+4\cos\theta.$$

m) Prove that the equation

$$6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$$

represents a pair of perpendicular straight lines.

n) Express $A = \begin{bmatrix} 10 & 20 \\ 30 & 50 \end{bmatrix}$ as a sum of a symmetric and a skew-symmetric matrix.

o) If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 3I = 0$. Hence find A^{-1}.

GROUP–B

[Marks : 20]

2. Answer any **four** questions: 5×4=20

a) Solve the following system of equation by using elementary row operations:

$$x + y = 4$$

$$y - z = 1$$

$$2x + y + 4z = 7$$

b) Solve the equation $x^3 - 18x - 35 = 0$.

c) Let (G, o) be a group and H and K are two subgroups of (G, o). Then prove that $H \cap K$ is a subgroup of (G, o).

Does the union of two subgroups of a group (G,o) form a subgroup of (G,o)? Explain.

d) Find the condition that one of the straight lines given by $ax^2+2hxy+by^2=0$ may coincide with one of the straight lines given by $a'x^2+2h'xy+b'y^2=0$.

e) Reduce the equation to its canonical form and determine the nature of the conic

$$3x^2+2xy+3y^2-16x+20=0.$$

f) If by a rotation of rectangular axes about the origin $(ax+by)$ and $(cx+dy)$ be changed to $(a'x'+b'y')$ and $(c'x'+d'y')$ respectively, show that $ad-bc=a'd'-b'c'$.

GROUP–C

[Marks : 20]

3. Answer any **two** questions: 10×2=20

a) i) Find the rank of the matrix

$$\begin{pmatrix} 1 & 0 & 3 \\ 4 & -1 & 5 \\ 2 & 0 & 5 \end{pmatrix}.$$

ii) Let (G, o) be a group and $a \in G$. Prove that $aG = G$, where $aG = \{a \circ g : g \in G\}$.

5+5

b) i) Examine whether the relation

$\rho = \{(a, b) \in Z \times Z : 3a + 4b$ is divisible by 7$\}$

is an equivalence relation on the set Z of all integers.

ii) If the straight lines $ax^2 - 2hxy + by^2 = 0$ form an equilateral triangle with the straight line $x\cos\alpha + y\sin\alpha = p$, then show that

$$\frac{a}{1-2\cos 2\alpha} = \frac{h}{2\sin 2\alpha} = \frac{b}{1+2\cos 2\alpha}.$$

5+5

c) i) Show that the straight line $r\cos(\theta - \alpha) = p$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$, if $(l\cos\alpha - ep)^2 + l^2\sin^2\alpha = p^2$.

ii) If $\tan(\theta + i\phi) = \tan\beta + i\sec\beta$ where θ, ϕ, β are real and $0 < \beta < \pi$. Show that $e^{2\phi} = \cot\frac{\beta}{2}$ and $\theta = n\pi + \frac{\pi}{4} + \frac{\beta}{2}$. 5+5

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