U.G. 1st Semester Examination - 2022

MATHEMATICS

[HONOURS]

Generic Elective Course [GE]

Course Code : MATH-H-GE-T-01

(Algebra and Analytical Geometry)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

The notations and symbols have their usual meanings.

GROUP–A

[Marks : 20]

1. Answer any **ten** questions: $2\times10=20$

a) Determine the values of z when $z^6=\sqrt{3}+i$.

b) Solve $x^n-1=0$, where n is a positive integer.

c) If a, b, c, d are the roots of $x^4+px^3+qx^2+rx+s=0$, find the value of $\sum a^2b$.

d) If z be a complex number, prove that $|z| \geq \frac{1}{\sqrt{2}}(|\mathrm{R}\,\mathrm{l}z| + |\mathrm{Im}\,z|)$.

e) Determine the inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$. Test whether the inverse is odd.

f) Show that the set $G = \{1, \omega, \omega^2\}$ of the cube roots of unity is a group with multiplicative composition.

g) Let $f: R \to R$ be defined by $f(x) = x^2$, $x \in R$. Can you find f^{-1}? Explain.

h) Let $(G, \circ)$ be a group and $a \in G$. Prove that $(a^{-1})^{-1} = a$.

i) Find the value of the determinant

$$\begin{vmatrix} 2022 & 2023 & 2024 \\ 2025 & 2026 & 2027 \\ 2028 & 2029 & 2030 \end{vmatrix}.$$

j) If A is an invertible matrix, show that transpose of A is also invertible.

k) The origin is shifted to the point (4, –5) without changing the directions of the axes. If the coordinates of a point be (1, 3) in the new system, find its coordinates in the old coordinate system.

l) Find the polar equation of the ellipse $\frac{x^2}{36}+\frac{y^2}{20}=1$ if the pole is at its right hand focus and the positive direction of the x-axis is the direction of the polar axis.

m) Determine whether the curve $3x^2-4xy+2y^2+4x-2y+1=0$ has infinitely many centres.

n) Find the values of b and f for which the equation $x^2+6xy+by^2+3x+2fy-4=0$ represents a central conic.

o) Find the point on the parabola $\frac{l}{r}=1-\cos\theta$ which has the smallest radius vector.

GROUP–B

[Marks : 20]

2. Answer any **four** questions: 5×4=20

a) Using De Moivre's theorem prove that

$$\tan 5\theta=\frac{5\tan\theta-10\tan^3\theta+\tan^5\theta}{1-10\tan^2\theta+5\tan^4\theta}.$$

b) If $x = \log\tan\left(\frac{\pi}{4}+\frac{y}{2}\right)$, show that

$$y = -i\log\tan\left(\frac{\pi}{4}+\frac{ix}{2}\right),$$

where x and y are real numbers.

c) Prove that the roots of the equation

$$\frac{1}{x-1}+\frac{2}{x-2}+\frac{3}{x-3}=x$$

are all real.

d) Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 5 \\ -1 & 4 & 0 \end{pmatrix}.$$

e) Let $(G, \circ)$ be a group. Prove that a non-empty subset H of G forms a subgroup of G if and only if $a \in H,\ b \in H \Rightarrow a \circ b^{-1} \in H$.

f) If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of parallel straight lines, show that the distance between them is $2\sqrt{\frac{g^2 - ca}{a(a+b)}}$.

GROUP–C

[Marks : 20]

3. Answer any **two** questions: $10\times2=20$

a) i) The distance of the point (h, k) from each of two given lines through the origin is d. Show that the equation of the lines is $(xk-yh)^2=d^2(x^2+y^2)$.

ii) Show that the locus of the point of intersection of perpendicular tangents to the conic $\frac{l}{r}=1-e\cos\theta$ is

$$r^2(1-e^2)-2ler\cos\theta-2l^2=0.$$

4+6

b) i) If every element, except the identity of a group be of order 2, then prove that the group is abelian.

ii) Consider the permutations

$$A=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 2 & 3 & 1 & 5 & 4\end{pmatrix} \text{ and } B=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 1 & 3 & 4 & 5 & 2\end{pmatrix}.$$

Determine AB. Verify whether AB is even.

6+4

c) i) Solve the following system by matrix method:

$$x_1 + 2x_2 - x_3 = 10$$

$$-x_1 + x_2 + 2x_3 = 2$$

$$2x_1 + x_2 - 3x_3 = 8$$

ii) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then find the equation whose roots are $\frac{\beta+\gamma}{\alpha^2}, \frac{\gamma+\alpha}{\beta^2}, \frac{\alpha+\beta}{\gamma^2}$.

5+5

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