U.G. 1st Semester Examination - 2022

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-01

(Calculus and Analytical Geometry)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

The notations and symbols have their usual meanings.

GROUP–A

[Marks : 20]

1. Answer any **ten** questions: $2\times10=20$

a) If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, $|x|<1$, show that $(1-x^2)y_2-3xy_1-y=0$.

b) Show that the radius of curvature of $r=a(\theta+\sin\theta)$ at $\theta=0$ is a.

c) Obtain the horizontal asymptote, if any, of the curve $y=e^{-x^2}$.

[Turn Over]

d) Show that (0, 0) is a double point on the curve $x^3 + y^3 - 3axy = 0$. Find the nature of the double point.

e) Find the point of inflexion of the curve $y - 3 = 6(x-2)^5$.

f) Find $\int_0^{\frac{\pi}{4}} \tan^5 x \, dx$.

g) Find the whole area of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$, bounded by its base.

h) Determine the volume of the part of the parabola $y^2=4ax$ bounded by the latus rectum revolves about the tangent at the vertex.

i) Determine the angle of rotation of the axes so that the equation $x+y+2=0$ may reduce to the form $ax+b=0$.

j) Find the equations of the straight line through the point (1, −2, 4) and parallel to the y-axis.

k) If the equation

$$ax^2 + by^2 + cz^2 - 4ax + 3by - 6cz + 5 = 0$$

represents a sphere, find its centre.

l) On the conic $r=\frac{21.}{5-2\cos\theta}$, find the point with the least radius vector.

m) Find the point on the parabola $\frac{l}{r}=1-\cos\theta$ which has the smallest radius vector.

n) The gradient of one of the straight lines of $ax^2+2hxy+by^2=0$ is twice of the other. Show that $8h^2=9ab$.

o) Evaluate $\int_0^4\int_0^1 xy(x-y)\,dy\,dx$.

GROUP–B

[Marks : 20]

2. Answer any **four** questions: $5\times4=20$

a) If $y=e^{ax}\cos bx$, then show that

$$y_n=\left(a^2+b^2\right)^{\frac{n}{2}}e^{ax}\cos(bx+n\theta),\text{ where }\cos\theta=\frac{a}{r},$$

$\sin\theta=\frac{b}{r}$ and $r^2=a^2+b^2$, $-\pi<\theta\le\pi$.

b) Show that the pedal equation of the parabola $r=\frac{2a}{1-\cos\theta}$ is $p^2=ar$.

c) The arc of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$, revolves about the x-axis. Show that the volume and the surface area of the solid generated are respectively $\frac{16}{105}\pi a^3$ and $\frac{6}{5}\pi a^2$.

d) Obtain a reduction formula for $\int \sec^n x\,dx$. Hence find the value of $\int \sec^6 x\,dx$.

e) Reduce the equation

$$7x^2 - 6xy - y^2 + 4x - 4y - 2 = 0$$

to its canonical form and find the nature of the conic.

f) Find the coordinates of the vertex, focus and the length of the latus rectum of the principal sections of the paraboloid given by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{c}.$$

GROUP–C

[Marks : 20]

3. Answer any **two** questions: $10\times2=20$

a) i) Show that the asymptotes of the curve $x^2y^2-4(x-y)^2+2y-3=0$ form a square of side 4 units.

ii) Find the equation of the generators of the hyperbolic paraboloid $16x^2-9y^2=4z$, passing through the point (1, 0, 4). 5+5

b) i) Find the nature and position of the singular points (if any) of the curve

$$x^2(x-y)+y^2=0.$$

ii) Show that the straight line $r\cos(\theta-\alpha)=p$ touches the conic $\frac{l}{r}=1+e\cos\theta$, if $(l\cos\alpha-ep)^2+l^2\sin^2\alpha=p^2$.

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c) i) Show that the area between the curve $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$, the x-axis and the ordinates at two points on the curve is equal to a times the length of the arc terminated by those points.

ii) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 2x - 4y + 5 = 0$, $x - 2y + 3z + 1 = 0$ is a great circle.

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