U.G. 1st Semester Examination - 2022

MATHEMATICS

[HONOURS]

Course Code: MATH-H-CC-T-02

(Algebra)

Full Marks: 60

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

GROUP-A

[Marks : 20]

1. Answer any ten questions:

 $2 \times 10 = 20$

a) Find the principal argument of $z = 1 + i \tan \frac{3\pi}{4}$.

Find the cube roots of -1.

Find the general value of i'.

d) Show that $\sin \left\{ i \log \left(\frac{a - ib}{a + ib} \right) \right\} = \frac{2ab}{a^2 + b^2}$.

Find the equation whose roots are the cube of the roots of the equation $x^3 + 3x^2 + 2 = 0$. If $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 3$, $x \in \mathbb{R}$, then find $f^{-1}(3)$ and $f^{-1}(7)$. $f^{-1}(x) \ge y$

- Give an example of a function which is neither one-one nor onto.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 1 & 3 & 4 \end{pmatrix}.$$

$$\gcd(33, 57) = 33s + 57t.$$

one-one nor onto.

Find the order of the permutation $\begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 \\
6 & 5 & 2 & 1 & 3 & 4
\end{pmatrix}.$ If $a \mid bc$ and gcd(a, b) = 1, then prove that $a \mid c$.

Find the integers s, t such that gcd(33, 57) = 33s + 57t.What is the remainder when 1! + 2! + 3! + ... + 100!is divided by 15? is divided by 15?

Find
$$\lambda$$
 such that $A = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & \lambda \end{pmatrix}$ is an

orthogonal matrix.

Find all real values of
$$\lambda$$
 for which the rank of

the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & \lambda \\ 5 & 7 & 1 & \lambda^2 \end{pmatrix}$ is 2. a $\begin{vmatrix} a & a & b \\ a & b & c \\ a & a & c \\$

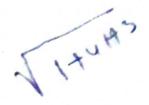
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(2)

- In a group (G, o), show that $a \circ b = a \circ c \Rightarrow b = c$ for all $a, b, c \in G$.
 - o) In any group G if $a^3 = e$ and $aba^{-1} = b^2$ for $a, b \in G$, then find the order of b.

GROUP-B

[Marks : 20]



Answer any four questions:

 $5 \times 4 = 20$

a) State De Moivre's theorem. If

$$z_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r} (r = 1, 2, 3, ...),$$

then prove that $z_1.z_2.z_3...\infty = -1$.

2 + 3

b) Show that
$$n^2 < 2^n \ \forall \ n \ge 5$$
.

5

- Solve by Cardan's method $x^3 12x + 65 = 0$. 5
- d) Define equivalence relation on a set S. Show that the relation

$$\rho = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a - b \text{ is divisible by 5}\}$$

is an equivalence relation on the set \mathbb{Z} of all integers.

Define a group. Give an example of finite and infinite group. Does the set Z of all integers

form a group under the operation defined as $a \circ b = a - b$ for all $a, b \in \mathbb{Z}$? Justify your answer. 2+1+2

Reduce the matrix
$$A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{pmatrix}$$
 to a

row reduced echelon form and hence find its rank.

4+1

GROUP-C

[Marks: 20]

Answer any two questions:

 $10 \times 2 = 20$

a) i) Show that the sum of 99th powers of the roots of the equation $x^5 - 1 = 0$ is zero.

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- ii) Apply Descarte's rule of signs to show that the equation $x^7 + 5x^4 - 3x + 1 = 0$ has at least four imaginary roots.
- iii) If one of the roots of the equation $x^3 + px^2 + qx + r = 0$ equals the sum of the other two, then prove that $p^3 + 8r = 4pq$.

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b) i) Let $f: A \to B$ and $g: B \to C$ be two mappings such that $g \circ f$ is injective and f is surjective. Prove that g is injective.

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- ii) In a group (G, \circ) , prove that $(a^{-1})^{-1} = a$ and $(a \circ b)^{-1} = b^{-1} \circ a^{-1}$ for all $a, b \in G$. 3
- iii) Show that the residue classes [1], [3], [5], [7] modulo 8 form a multiplicative group.
- c) i) Investigate for what values of λ and μ the following equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

have no solution, a unique solution and an infinite number of solutions.

Obtain the normal form under congruence and find the rank, signature of the

symmetric matrix
$$\begin{pmatrix} 2 & 4 & 3 \\ 4 & 6 & 3 \\ 3 & 3 & 1 \end{pmatrix}$$
. 5

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