U.G. 1st Semester Examination - 2019

MATHEMATICS

(PROGRAMME)

Course Code : MATH(G)CC-01-T

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.
Symbols and notations have their usual meanings.

1. Answer any **ten** questions : 2×10=20

i) Find $\lim_{x\to 0} x\sin\frac{1}{x}$.

ii) Find the nature of discontinuity of the given function

$$f(x)=\sin\frac{1}{x}, \quad x\neq 0$$
$$=0, \quad x=0.$$

iii) Verify Rolle's theorem for the function $f(x)=e^x,\ x\in[0,1]$.

iv) Find $\frac{dy}{dx}$ if $x=a\cos^3 t$ and $y=b\sin^3 t$.

v) Find from definition, the partial derivative of the function $f(x,y)=x^{\frac{3}{2}}e^y$ w.r. to x at the point (1, 2).

vi) Find $Rf'(1)$ of the function $f(x)=|x-1|$.

vii) Find the radius of curvature of any point on the curve $r=a(1+\cos\theta)$.

viii) Examine the differentiability of

$$f(x)=|x| \text{ at } x=0.$$

ix) Find the asymptotes of the curve $x^2y^2-a^2(x^2+y^2)-a^3(x+y)+a^4=0$.

x) Verify Rolle's theorem for the function $f(x)=x\sqrt{a^2-x^2}$ in [0, 1].

xi) Investigate the continuity of the function

$$f(x)=\frac{|x|}{x}, \text{ when } x \neq 0$$
$$=-1, \quad \text{when } x=0 \quad \text{at } x=0$$

xii) For $0 \leq a < x$, prove that $x^a < a^x$

xiii) If $u=xf\left(\frac{y}{x}\right)+g\left(\frac{y}{x}\right)$, then show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=xf\left(\frac{y}{x}\right).$$

xiv) Find the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

xv) $f(x)=\frac{1-\cos x}{x^2}, x \neq 0$

$$=1, \quad x=0$$

Check whether the function f(x) is continuous at x = 0.

2. Answer any **four** questions: $5\times4=20$

i) Prove that if the curves $ax^2+by^2=1$ and $Ax^2+By^2=1$ intersect at right angles, then $\frac{1}{A}-\frac{1}{a}=\frac{1}{B}-\frac{1}{b}$. 5

ii) State Leibnitz's theorem for the nth derivative of product of two functions. If $y=e^{m\sin^{-1}x}$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$. 2+3

iii) Find $\lim_{x\to0}\frac{(1+x)^{\frac{1}{x}}-e}{x}$. 5

iv) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, then prove that

$$x^2\frac{\partial^2u}{\partial x^2}+2xy\frac{\partial^2u}{\partial x\partial y}+y^2\frac{\partial^2u}{\partial y^2}=\sin 2u\left(1-4\sin^2u\right).$$

5

v) Discuss the asymptotes of the curve $y=\frac{3x}{2}\log\left(e-\frac{1}{3x}\right)$. 5

vi) Show that $f(x,y)=\frac{xy^2}{x^2+y^2}$, if $x\neq0$

$=0$, if $x=0$

possesses 1st order partial derivatives at (0, 0). 5

Answer any **two** questions: $10\times2=20$

3. a) Trace the curve $r=a(1-\cos\theta)$. 5

   b) Find the angle of intersection of the parabolas $y^2=ax$ and $x^2=ay$. 5

4. a) State and prove Lagrange's mean value theorem. 5

   b) Show that $\frac{x}{1+x}<\log(1+x)<x,\ \forall\ x>0$. 5

5. a) Find the asymptotes of

$$(y+x+1)(y+2x+2)(y+3x+3)(y-x)+x^2+y^2-8=0.$$

5

   b) Show that normal at any point θ of the curve $x=a\cos\theta+a\theta\sin\theta,\ y=a\sin\theta-a\theta\cos\theta$ is at a constant distance from the origin. 5

6. a) Find the radius of curvature of $y=xe^{-x}$ at its maximum point. 5

   b) Using Caucy's Mean Value Theorem for the functions $f(x)=e^x$ and $g(x)=e^{-x}$ defined in $[a,b]$ prove that $c\in(a,b)$ is the arithmetic mean of a and b. 5
