U.G. 6th Semester Examination - 2022

MATHEMATICS

[PROGRAMME]

Discipline Specific Elective (DSE)

Course Code : MATH-G-DSE-T-02(A)&(B)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Symbols and Notations have their usual meanings.

Answer all the questions from selected Option.

OPTION–A

MATH-G-DSE-T-02A

(Linear Programming)

1. Answer any **ten** questions : 2×10=20

a) What is meant by a basic feasible solution for a system of equations AX=b of m equations in n unknowns n>m?

b) Define a convex set with example.

c) Prove that the vectors (1,1,0), (1,–l,0) and (0,0,1) form a basis for E^3.

d) State fundamental theorem of L.P.P.

e) What is the value of M in Big - M method?

f) What are the assumptions made in the theory of games?

g) Define Mixed Strategy.

h) Define saddle point in a game theory.

i) When artificial variables are used in a L.P.P.?

j) Prove that $X=\{(x_1,x_2)\,|\,x_1 \le 5, x_2 \ge 3\}$ is a convex set.

k) Write down the standard form of the given LPP

Max $z = 6x_1 - 2x_2$

subject to $2x_1 - x_2 \le 2$

$x_2 \ge 4$

$x_1, x_2 \ge 0$

l) State the optimality condition for existing a basic feasible solution of a LPP.

m) Prove that dual of the dual is the primal.

n) Define a two-person zero-sum game.

o) Define pay-off matrix.

2. Answer any **four** questions: 5×4=20

a) Solve by simplex method

Max $z = 3x_1 + 2x_2$

subject to $x_1 + x_2 \ge 1$

$2x_1 + x_2 \le 4$

$5x_1 + 8x_2 \le 15$

$x_1, x_2 \ge 0$

b) Solve by simplex method

Max $z = 3x_1 + x_2 + 3x_3$

subject to $2x_1 + x_2 + x_3 \le 2$

$x_1 + 2x_2 + 3x_3 \le 5$

$2x_1 + 2x_2 + x_3 \le 6$

$x_1, x_2, x_3 \ge 0$

c) Obtain the dual problem of the following LPP

Max $z = 2x_1 + 5x_2 + 6x_3$

subject to $5x_1 + 6x_2 - x_3 \le 3$

$-2x_1 + x_2 + 4x_3 \le 4$

$x_1 - 5x_2 + 3x_3 \le 1$

$-3x_1 - 3x_2 + 7x_3 = 6$

$x_1, x_2 \ge 0$

and x_3 is unrestricted in sign.

d) Solve the following transportation problem.

	D_1	D_2	D_3	D_4	a_i
O_1	2	3	11	7	6
O_2	1	0	6	1	1
O_3	5	8	15	9	10
b_j	7	5	3	2	

e) Solve the following assignment problem.

	I	II	III	IV	V
A	62	78	50	101	82
B	71	84	61	73	59
C	87	92	111	71	81
D	48	64	87	77	80

f) For the following Pay-off table, transform the zero sum game into an equivalent LPP.

	Player B		
Player A	9	1	4
	0	6	3
	5	2	8

3. Answer any **two** questions: 10×2=20

a) i) Solve the following transportation problem.

	D_1	D_2	D_3	D_4	a_i
O_1	3	7	2	1	11
O_2	9	4	7	3	20
O_3	10	2	8	3	35
b_j	10	5	21	30	

ii) Solve the following assignment problem.

	I	II	III	IV
A	42	35	28	21
B	30	25	20	15
C	30	25	20	15
D	24	20	16	12

b) i) Use graphical method to solve the following game.

	Player B	
Player A	2	4
	2	3
	3	2
	-2	6

ii) Solve the game whose pay-off matrix is given by

Player B

Player A				
	3	2	4	0
	2	4	2	4
	4	2	4	0
	0	4	0	8

c) Products X and Y are to be blended to produce a mixture that contains at least 30% A and 30% B. Product X is 50% A and, 4% B and costs Rs.10 gal; Product Y is 20% A and 10% B and costs Rs.2/gal. Formulate and solve the model to be used to determine a minimal-cost blend.

OPTION–B

MATH-G-DSE-T-02B

(Numerical Methods)

1. Answer any **ten** questions : 2×10=20

a) Find the relative error the computation of $y = x^3 + 3x^2 - x$, for $x = \sqrt{2}$, taking $\sqrt{2} = 1.414$.

b) If $y = 6x^4 - 5x$, find the percentage error in $y(1)$ when the error in x is 0.03

c) Find the k^{th} difference of a polynomial of degree k.

d) Show that $\Delta\{\log f(x)\} = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$.

e) Prove that
$y' = \frac{dy}{dx} = \frac{1}{h}\left[\Delta y - \frac{1}{2}\Delta^2 y + \frac{1}{3}\Delta^3 y - \frac{1}{4}\Delta^4 y + \cdots\right]$

f) Show that the operators E, Δ and ∇ commute.

g) If $f(x) = e^{\alpha x + \beta}$, prove that $f(0)$, $\Delta f(0)$ and $\Delta^2 f(0)$ are in G.P.

h) If $f(-2) = f(3) = 7$ and $f(0) = 1$, find $f(10)$.

i) For the eqation $x^3 + x^2 - 1 = 0$, construct a fixed point iteration form $x = g(x)$ so that the method converges in the interval [0, 1].

j) Find the minimum number of iterations required to attain an accuracy of 0.001 in an interval [1, 2] using bisection method.

k) Find the iterative formula for Newton-Raphson method to find the square root of $\sqrt{m}$.

l) Using Newton-Raphson method obtain the root of $x^3 - 8x - 4 = 0$ correct upto two decimal places (Take the initial approximation as $x_0 = 0$).

m) Use Trapezoidal rule to evaluate $\int_0^6 y(x)\, dx$ for the data.

x	0	1	2	3	4	5	6
y	0.146	0.161	0.167	0.19	0.204	0.217	0.23

n) The Trapezoidal rule applied to $\int_1^3 f(x)\, dx$ gives the value 8 and the Simpson's one-third rule gives the value 4. Find $f(2)$.

o) Apply Runge-Kutta method of fourth order to find an approximate value of $y(0,2)$, given that $\frac{dy}{dx} = x + y$ and $y(0) = 1$.

2. Answer any **four** questions: 5×4=20

a) A function $f(x)$ defined in [0, 1] in such a way that $f(0) = 0$, $f\left(\frac{1}{2}\right) = -1$ and $f(1) = 0$. Find the interpolating polynomial $p(x)$ approximating $f(x)$. If $|\frac{d^3 f}{dx^3}| \leq 1$ for $0 \leq x \leq 1$, show that $|f(x) - p(x)| \leq \frac{1}{2}$.

b) Prove that the Lagrange's interpolation formula can be put in the form

$$P_n(x) = \sum_{r=1}^{n} \frac{\phi(x) f(x_r)}{(x - x_r)\phi'(x_r)}, \text{ where } \phi(x) = \prod_{r=0}^{n} (x - x_r).$$

c) Find a real root of $x^3 - 8x - 4 = 0$ between and 4 by using Newton-Raphson method correct upto four decimal places.

d) Find the largest eigenvalue and the corresponding eigenvector of the following matrix correct upto four significant figures

$$A = \begin{pmatrix} 9 & 10 & 8 \\ 10 & 5 & -1 \\ 8 & -1 & 3 \end{pmatrix}$$

Take the initial approximate eigen vector $(X_0 = (1, 0, 0)^T$.

e) Apply Gauss-Seidel iteration method solve the system of equation:

$$\begin{aligned} 8x - y + z &= 18 \\ x + y - 3z &= -6 \\ 2x + 5y - 2z &= 3 \end{aligned}$$

Continue iterations until two successive approximations are identical when rounded to three significant digits.

f) Find $y(4.4)$, by Euler's Modified Method, taking h=0.2, from the differential equation $\frac{dy}{dx} = \frac{2 - y^2}{5x}$, $y(4) = 1$.

3. Answer any **two** questions: 10×2=20

a) i) If n be a positive integer, prove that

$$\Delta^r x^{(-n)} = (-1)^r n(n+1)(n+2)....(n+r-1)h^r x^{-(n+r)}$$

ii) From the following table find $f(8.2)$ by using Newton's forward interpolation formula

x	8	8.5	9.5	10
$f(x)$	50	57	17	78

b) i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 3$ for the function $y = f$ (x) given in the table.

x	1	2	3	4	5	6
y	2.7183	3.3210	4.0552	4.9530	6.0496	7.3891

ii) Let $f(x)$=0 has real root in an interval [a, b] where $f(x)$=0 can be rewritten as x=$g(x)$. Then prove that the function y=$g(x)$ has a fixed point $\bar{x} = g(\bar{x})$ in [a, b] if $|g(x')| \leq c < 1$, for x in $[a, b]$.

c) i) If $x_{n+1} = \alpha x_n\left(3 - \frac{x_n^2}{a}\right) + \beta x_n\left(1 + \frac{a}{x_n^2}\right)$ has a third order convergence to $\sqrt{a}$, then show that $\beta = 3\alpha$.

ii) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\, dx$ by Simpson's one-third rule, taking 6 intervals.

d) i) Use Euler's method to approximate the solution of $\frac{dx}{dt} = tx^3 - x$ $(0 \leq t \leq 1)$, $x(0) = 1$ over the interval [0, 1] using four steps.

ii) Evaluate $y(1)$ from the differential equation $\frac{dy}{dx} = x^2 + y$ with $y(0)$=1, taking h=0.5 by the fourth oredr Runge-Kutta method and hence, compare it to original solution.

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