U.G. 6th Semester Examination - 2021

MATHEMATICS

[PROGRAMME]

Discipline Specific Elective (DSE)

Course Code : MATH-G-DSE-T-02(A)&(B)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Symbols and Notations have their usual meanings.

Answer all the questions from selected Option.

OPTION–A

MATH-G-DSE-T-02A

(Linear Programming)

1. Answer any **ten** questions : 2×10=20

a) Solve Max $Z = 5x_1 - 3x_2$

subject to $3x_1 + 5x_2 \le 15,\ x_1, x_2 \ge 0$

b) What is extreme point?

c) Show that $X = \{x : |x| < k, k > 0\}$ is a convex set.

d) What basic feasible solution of an LPP?

e) When an LPP has alternative solutions?

f) What is artificial variables?

g) When Big M method is used to solve an LPP.

h) Convert the LPP

Max $Z = CX,\ X \ge 0$

subject to $AX \le b$ into its dual form.

i) Find initial basic feasible solution by North-West corner method.

	D_1	D_2	D_3	D_4	
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
	5	8	7	14	

j) Where a transportation problem is said to be unbalanced?

k) Write down mathematical form of assignment problem.

l) What is saddle point of a game?

m) State fundamental theorem of game theory.

n) Find the value of the game with pay off matrix.

5	1
3	4

o) Find the minimum number of non-basic cells in an m×n transportation problem.

2. Answer any **four** questions: 5×4=20

a) Solve the following LPP by graphical method

Minimize $Z = 20x_1 + 10x_2$

subject to $x_1 + 2x_2 \le 40$

$3x_1 + x_2 \ge 30$

$4x_1 + 3x_2 \ge 60$

$x_1, x_2 \ge 0$

b) Solve the following LPP by simplex method.

Max $Z = 3x_1 + 2x_2$

subject to $2x_1 + x_2 \le 40$

$x_1 + x_2 \le 24$

$2x_1 + 3x_2 \le 60$

$x_1, x_2 \ge 0$

c) Prove that the dual of the dual is the primal.

d) Solve the following transportation problem:

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	3	4	6	8	8	20
O_2	2	10	0	5	8	30
O_3	7	11	20	40	3	15
O_4	1	0	9	14	16	13
b_j	40	6	8	18	6	

e) Find the optimum assignment to find the maximum profit for the assignment problem with the profit matrix.

	I	II	III	IV
A	7	5	4	3
B	8	2	6	4
C	5	3	2	1
D	5	4	1	8

f) Solve graphically gmae whose pay-off matrix is

		B		
		B_1	B_2	B_3
A	A_1	1	3	11
	A_2	8	5	2

3. Answer any **two** questions: 10×2=20

a) i) Solve the LPP

Maximize $Z = x_1 + 5x_2$

subject to $3x_1 + 4x_2 \le 6$

$x_1 + 3x_2 \ge 3$

$x_1, x_2 \ge 0$

ii) Prove that a hyper plane is a convex set.

b) i) Find the dual of the following LPP.

Maximize $Z = 6x_1 + 4x_2 + 6x_3 + x_4$

subject to $4x_1 + 4x_2 + 4x_3 + 8x_4 = 21$

$3x_1 + 17x_2 + 80x_3 + 2x_4 \leq 48$

$x_1, x_2 \geq 0$, x_3, x_4 are unrestricted in sign.

ii) Prove that the transportation problem always has a feasible solution.

c) i) Solve the game by simplex method.

B

		B_1	B_2	B_3
	A_1	9	1	4
A	A_2	0	6	3
	A_3	5	2	8

ii) Find the values of a so that the game with the following pay off matrix is strictly determinable

B

		I	II	III
	I	a	5	2
A	II	−1	a	−8
	III	−2	3	a

OPTION–B

MATH-G-DSE-T-02B

(Numerical Methods)

1. Answer any **ten** questions : 2×10=20

a) Find the relative and percentage errors in an approximate representation of $\pi = 3.14159$ by $\frac{22}{7}$.

b) Prove that $(1+\Delta)(1-\nabla) = 1$, where Δ and ∇ are forward and backward difference operators respectively.

c) Evaluate $\Delta^3[(1-x)(1-2x)(1-3x)]$, the interval of differencing being 1.

d) If $f(x) = e^{ax+b}$, prove that $f(0), \Delta f(0)$ and $\Delta^2 f(0)$ are in G.P.

e) Find a function whose first forward difference is e^x.

f) Write the Lagrange's interpolating polynomial for the three points (x_0, y_0), (x_1, y_1) and (x_2, y_2).

g) Find $f(0.2)$ from the following table:

x	0	0.5	1
f(x)	3.25	4.17	5.03

h) Derive $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ for determining the the square root of $a > 0$, using Newton-Raphson method.

i) Using Newton-Raphson method obtain the root of $x^3 - 5x + 1 = 0$ correct upto two decimal places (Take the initial approximation as $x_0 = 0$).

j) Give the graphical interpretation of the method of false position.

k) For the equation $x^3 + x^2 - 1 = 0$, construct a fixed point iteration form $x = g(x)$ so that the method converges in the interval [0, 1].

l) A particle is moving along a straight line. The displacement x at time t is given as follows:

t	0	1	2	3
x	5	8	12	17

Find the velocity of the particle at $t = 3$.

m) Show that the iteration scheme $x_{n+1} = \frac{\sin x_n}{x_n}$ converges for all $x_n \geq 2$.

n) $f(x)$ is given by:

x	0	0.5	1
f(x)	1	0.8	0.5

Using Trapezoidal rule find the value of $\int_0^1 f(x)\, dx$.

o) Using Euler's method, find $y(0.05)$, where $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$ and $h = 0.05$.

2. Answer any **four** questions: 5×4=20

a) Use Lagrange's interpolation to find $f(3)$ from the following table :

x	0	1	2	4	5	6
f(x)	1	14	15	5	6	19

b) Use Newton's forward interpolation formula to establish the formula

$$\left(\frac{d^2y}{dx^2}\right)_{x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \frac{137}{180}\Delta^6 y_0 + \cdots\right]$$

c) Apply Gauss-Seidel iteration method to solve the system of equations:

$20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

Perform two iterations with zero vector as an initial approximation.

d) Use the power method to find the largest eigenvalue in magnitude and the corresponding eigenvector of the matrix

$$A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$$

Take the initial approximate eigenvector $v_0 = (1, 1, 1)^T$ and perform 3 iterations.

e) The initial value problem $\frac{dy}{dt} = t^2 + y,\ y(1) = 2$ is given. Find $y(1.4)$ for $h = 0.1$ and $h = 0.2$ using Euler's method.

f) Find the inverse of the matrix

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

using LU decomposition method and taking $u_{11} = u_{22} = u_{33} = 1$.

3. Answer any **two** questions: 10×2=20

a) i) Define k^{th} order difference of a function $f(x)$. Prove that for equally spaced interpolating points $x_i = x_0 + ih$ $(h > 0$ and $i = 1, 2 \ldots, n)$,

$$\Delta^k f(x) = \Sigma_{i=0}^{k} (-1)^i \begin{pmatrix} k \\ i \end{pmatrix} f[x + (k - i)h].$$

ii) Find an interpolating polynomial that fits the data:

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

Hence interpolate at x=3.

b) i) Find $f'(5)$ from the following table:

x	0	2	3	4	7	9
f(x)	4	26	58	112	466	922

ii) Explain Bisection method to solve ane quation of the form $f(x)$=0. Why bisection method is not applied to evaluate a double root of an equation.

c) i) Establish Trapezoidal rule by using Newton's forward interpolation formula. Hence, derive the composite form of Trapezoidal rule.

ii) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{1 - 0.162 \sin^2 x}\, dx$ by Simpson's 1/3 rd rule taking 10 intervals.

d) i) Use Euler's modified method to find the value of $f(0.1)$, where $y(x)$ is given by $\frac{dy}{dx} = y - \frac{2x}{y},\ y(0) = 1.$

ii) Using the fourth order Runge-Kutta method, find the approximate value of $y(0.4)$ for the initial value problem $\frac{dy}{dx} = x^2 + xy - 2,\ y(0) = 0,$ with step size $h = 0.2$

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