U.G. 6th Semester Examination - 2021

MATHEMATICS

[PROGRAMME]

Skill Enhancement Course (SEC)

Course Code : MATH-G-SEC-T-04(A)&(B)

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions from selected Option.

OPTION–A

MATH-G-SEC-T-04A

(Probabiity and Statistics)

1. Answer any **five** questions : 2×5=10

a) Show that $P(\Phi)=0$, where Φ is the empty set.

b) Write axiomatic definition of probability.

c) Suppose that X is a continuous random variable with probability density function

$$f(x)=kx, \quad 0<x<2,$$
$$=0, \quad \text{elsewhere.}$$

Find the value of *k*.

[Turn over]

d) Find the expectation *E(X)*, where *X* be uniformly distributed over the interval [a, b] .

e) Find moment generating function for Poisson distribution.

f) Define joint probability density function both in case of discrete and continuous random variables.

g) Let *(X, Y)* be two dimensional random variables and suppose that X and Y are independent. Then prove that *E (XY)= E(X) E (Y)*.

h) The probability density function of a random variable is defined by

$$f(x)=20x^3(1-x), \; 0<x<1$$
$$=0, \text{ elsewhere}$$

Evaluate $P(X \le \frac{2}{3})$.

2. Answer any **two** questions: 5×2=10

a) Two dice are thrown. Find the probability that the sum of the faces equals or exceeds 10 and hence find the probability that the sum of the faces less than 10.

b) What is random variable? Define cumulative distribution function of a random variable X both in case of discrete and continuous random variables. Show that it is a non-decreasing function.

c) Find the moment generating function of uniform distribution in $(-a, a)$, $a>0$. Hence find moments of order 3 about the origin.

d) Let the probability density function of a random variable X is given by

$$f(x) = c(4x - 2x^2), 0 < x < 2$$
$$= 0, \qquad \text{otherwise}$$

find the value of c and $P\left\{\frac{1}{2} < x < \frac{3}{2}\right\}$.

3. Answer any **two** questions: 10×2=20

a) i) The joint density function of X and Y is given by

$$f(x, y) = 6xy(2 - x - y),\ 0 < x < 1, 0 < y < 1$$
$$= 0 \text{ otherwise}$$

Compute conditional expectation of X given that $Y = y$ where $0 < y < 1$. 5

ii) Suppose that $P(x, y)$, the joint probability mass function of X and Y is given by $P(1,1) = 0.5, P(1,2) = 0.1, P(2,1) = 0.1, P(2,2) = 0.3$. Calculate the conditional probability mass function of X given that Y=1. 5

b) i) Let X be a continuous random variable with probability density function f given by

$$f(x) = ax, 0 \le x \le 1,$$
$$= a, 1 \le x \le 2$$
$$= -ax + 3a, 2 \le x \le 3,$$
$$= 0, \quad \text{elsewhere.}$$

Determine the constant a. Determine the cumulative distribution function F and sketch its graph. 2+3+2

ii) What is the probability of getting an odd-sum when two fair dice are thrown? 3

c) i) There are 3 urns containing white and black balls. The first urn contains 2 white and 3 black balls, the second urn 3 white and 5 black balls and the third urn 5 white and 2 black balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball drawn is white. 5

ii) Prove that $F(\infty) = 1$ and $F(x)$ is discontinuous at $x = a$ if and only if $P(x = a) \ne 0$, where $F(x)$ is the distribution function. 2+3

d) i) What do you mean by exponential distribution? Calculate its cumulative distribution function $F(x)$. 5

ii) If X is uniformly distributed over (0, 10), calculate the probability that (A) $X<3$, (B) $X>7$, (C) $1<X<6$. 5

OPTION–B

MATH-G-SEC-T-04B

(Boolean Algebra)

1. Answer any **five** questions : 2×5=10

a) What do you mean by an order set? Give an example.

b) Give an example of a poset which has exactly one maximal element but does not have a greatest element.

c) Let $(\mathbb{R}, \leq)$ be the poset of all real numbers and let $A = \{x \in \mathbb{R} : x^3 < 3\}$. Is there any upper bound of A? Justify your answer.

d) Define order isomorphism with an example.

e) Let $P = Q = (\mathbb{Z}; \leq)$. Is the map $\phi : P \to Q$ defined by $\phi(x) = x + 1$ order preserving? Justify your answer.

f) If L is a lattice and $a, b, c \in L$ then show that $(a \vee b) \vee c = a \vee (b \vee c)$.

g) What is a distributive lattice? Let M be a non empty set. Is $(\mathcal{P}(M), \cap, \cup)$ a distributive lattice?

h) In a Boolean algebra $(B, +, ., ')$, if $a + b = 0$ then prove that $a.b' = 0$ and $a'.b = 0$.

2. Answer any **two** questions: 5×2=10

a) Let P be a set on which binary relation $\leq$ is defined by $x \leq y \Leftrightarrow (x < y \text{ or } x = y)$.

Prove that $(P; \leq)$ is a poset. 5

b) In any lattice L, prove that

$$((x \wedge y) \vee (x \wedge z)) \wedge ((x \wedge y) \vee (y \wedge z)) = x \wedge y$$

for all $x, y, z \in L$. 5

c) For all x, y in a Boolean algebra $((B, +, ., ')$, prove that $(x + y)' = x'.y'$ and $(x.y)' = x' + y'$. 5

3. Answer any **two** questions: 10×2=20

a) i) Define lattice homomorphism. If L, K are lattices and $f : L \to K$ a lattice homomorphism, then prove that f is order preserving. 1+4

ii) Let L, K be lattices and $f : L \to K$. Prove that f is a lattice isomorphism if and only if it is an order isomorphism. 5

b) i) Define a complete lattice. Prove that $(\mathcal{P}(X);\subseteq)$ is a complete lattice where X is a non empty set. 1+4

ii) Let X be a set and $\mathcal{L}$ a family of subsets of X ordered by inclusion such that $\bigcap_{i\in I} A_i \in \mathcal{L}$ for every non empty family $\{A_i\}_{i\in I} \subseteq \mathcal{L}$ and $X \in \mathcal{L}$. Prove that $\mathcal{L}$ is a complete lattice with

α) $\bigwedge_{i\in I} A_i = \bigcap_{i\in I} A_i$

β) $\bigvee_{i\in I} A_i = \cap\{B \in \mathcal{L} : \bigcup_{i\in I} A_i \subseteq B\}$. 5

c) i) Let a Boolean function f be defined by $f(x,y) = xy' + x'y$. Find the disjunctive normal form of $f(x, y)$.

ii) Draw the circuit which represents the Boolean expression $xy + yz + zx$.

5+5

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