

U.G. 6th Semester Examination - 2022

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-03A

(Fuzzy Set Theory)

Full Marks : 60

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20
- i) If $A = [-3, 2]$ and $B = [7, 9]$, find $B - A$.
 - ii) Find the distance between $[2, 6]$ and $[5, 8]$.
 - iii) Find the support of the fuzzy number
 $A(x) = -x^2 + 6x - 8$.
 - iv) Give an example of a normal fuzzy set.
 - v) Define fuzzy number.
 - vi) Find all strong α -cuts of the fuzzy set
 $A = \frac{0.5}{x_1} + \frac{0.7}{x_2}$.

- vii) When a fuzzy set is said to be convex?
 - viii) Give an example of a non-convex fuzzy set.
 - ix) Find standard fuzzy complement of the set
 $A = \frac{0.3}{x_1} + \frac{0.9}{x_2}$.
 - x) Show that $A \cup A^c = X$ does not always hold for fuzzy set, A defined on X .
 - xi) Give an example of a symmetric binary fuzzy relation.
 - xii) Find the relationship between max-min composition and relational join.
 - xiii) From the nature of a relational matrix how the relation can be identified as reflexive and symmetric?
 - xiv) Give an example of a fuzzy similarity relation with justifications.
 - xv) State true or false with justifications:
Intersection of two normal fuzzy sets is normal.
2. Answer any **four** questions: 5×4=20
- i) Check whether distributive property for interval numbers holds or not.
 - ii) Find the standard max-min composition $P \circ Q$ of the following two binary fuzzy relations P and Q :

[Turn Over]

$$P = \begin{matrix} & a & b & c \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0.6 & 0.2 & 0 \\ 0.3 & 0 & 0 \\ 0 & 0.9 & 0.7 \\ 0.5 & 0 & 0.2 \end{bmatrix} \end{matrix} \text{ and } Q = \begin{matrix} & \alpha & \beta \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 0.1 & 0.2 \\ 0.8 & 0 \\ 0.5 & 0.6 \end{bmatrix} \end{matrix}$$

iii) Find the transitive max-min closure of the fuzzy relation:

$$R = \begin{bmatrix} 0.7 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0.8 & 1 \\ 0 & 0.4 & 0 & 0.4 \\ 0 & 0.4 & 0.8 & 0 \end{bmatrix}$$

iv) Represent the following fuzzy set in terms of union of special fuzzy sets using α -cuts:

$$A = 0.8/x_1 + 0.5/x_2 + 0.4/x_3 + 1/x_4 + 0.3/x_5$$

v) Establish the proper inclusion relationship between $\bigcap_{i \in I} A_i^{\alpha}$ and $A^{\alpha}(\bigcap_{i \in I} A_i)$. Also show that the reverse inclusion of the established relation does not hold, in general.

vi) If R is a reflexive binary fuzzy relation, prove that $R \subset R \circ R$.

3. Answer any **two** questions: 10×2=20

i) Using extension principle define $f(A)$. Find the relationship between ${}^{\alpha}f(A)$ and $f({}^{\alpha}A)$. Justify whether those are equal. 2+5+3

ii) Let A, B be two fuzzy numbers whose membership functions are given by

$$A(x) = \begin{cases} \frac{x+2}{2} & \text{if } -2 \leq x \leq 0 \\ \frac{2-x}{2} & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

and

$$B(x) = \begin{cases} \frac{x-2}{2} & \text{if } 2 \leq x \leq 4 \\ \frac{6-x}{2} & \text{if } 4 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the fuzzy number $B - A$ and AB .

4+6

iii) Show that the transitive closure of the following fuzzy relation represents an equivalence relation. Find partition tree of that equivalence relation. 6+4

$$R = \begin{bmatrix} 1 & 0 & 0.9 & 0 & 0.7 & 0.9 \\ 0 & 1 & 0 & 0.7 & 0 & 0.6 \\ 0.9 & 0 & 1 & 0.9 & 0 & 0 \\ 0 & 0.7 & 0.9 & 1 & 0 & 0 \\ 0.7 & 0 & 0 & 0 & 1 & 0.7 \\ 0.9 & 0.6 & 0 & 0 & 0.7 & 1 \end{bmatrix}$$