U.G. 6th Semester Examination - 2021

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-03A

(Fuzzy Set Theory)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Find magnitude of the interval number, $A=[-5,-1]$. How it differs from its width?

b) Find $A \bullet B^{-1}$, where the interval numbers are given by $A=[-1,2]$ and $B=[3,4]$.

c) Find the distance between two interval numbers $A=[2,7]$ and $B=[5,8]$.

d) Define two level interval numbers.

e) Show by an example that standard intersection of two normal fuzzy sets may not be normal.

f) For any $A \in \mathcal{F}(X)$ and $\alpha, \beta \in [0,1]$, show that ${}^{\alpha}A \supset {}^{\beta+}A$.

g) Give an example of a fuzzy number.

h) Express a discrete fuzzy set which represents the standard fuzzy complement of $A(x)=\frac{x}{x+2}$ defined on $\{0,1,2,3,4,5\}$.

i) Stating the reason check the normality of the fuzzy set A as defined in Q. 1. (h) above.

j) Using the concept of triangular fuzzy number, describe real numbers 'about – 2'.

k) Using standard fuzzy union and standard fuzzy intersection, find $A \cup B$ and $A \cap B$, where

$$A=\frac{0.2}{0}+\frac{0.5}{3}+\frac{0.7}{4}+\frac{0.9}{6} \text{ and}$$

$$B=\frac{0.3}{0}+\frac{0.4}{3}+\frac{0.6}{4}+\frac{0.8}{6}.$$

l) Give an example of symmetric fuzzy relation.

m) When a fuzzy binary relation is called max-min transitive?

[Turn Over]

n) Check whether $R=\begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix}$ represents a fuzzy similarity relation or not.

o) Give an example of a fuzzy tolerance relation.

2. Answer any **four** questions: 5×4=20

a) For any three interval numbers, check whether the distributive law is valid or not.

b) For the interval numbers, $A=[2,4]$, $B=[3,6]$ and $C=[1,5]$, verify the triangle inequality $d(A,C)+d(B,C)\ge d(A,B)$, where d stands for distance.

c) Define interval valued fuzzy sets. Compare ordinary fuzzy sets and interval valued fuzzy sets.

d) Find strong $\alpha-$cut, height and support of the following fuzzy set

$$A(x)=\begin{cases} 0 & if \;\; x\le 5 \;\; or \;\; x\ge 50 \\ \dfrac{x-5}{15} & if \;\; 5\le x\le 20 \\ \dfrac{50-x}{30} & if \;\; 20\le x\le 50 \end{cases}$$

e) For any two fuzzy sets, A and B defined on a universal set, prove that $|A|+|B|=|A\cap B|+|A\cup B|$, where $\cap$ and $\cup$ are standard fuzzy intersection and union, respectively and $|A|$ represents scalar cardinality of A.

f) Let A and B be fuzzy sets defined on the universal set $\mathbb{Z}$ whose membership functions are given by

$A(x)=0.5/(-2)+1/0+0.5/1+0.3/3$ and

$B(x)=0.5/2+1/3+0.5/6+0.3/8$.

Also, let a function $f:\mathbb{Z}\times\mathbb{Z}\to\mathbb{Z}$ be defined for all $x_1, x_2\in\mathbb{Z}$ by $f(x_1,x_2)=x_1+x_2$. Calculate $f(A,B)$.

3. Answer any **two** questions: 10×2=20

a) i) State and prove second decomposition theorem.

ii) A fuzzy binary relation R is defined on sets $X=(1,2,\ldots,100)$ and $Y=(50,51,\ldots,100)$ by the membership function,

$$R(x, y) = \begin{cases} 1 - \frac{x}{y} & \text{for } x \le y \\ 0 & \text{otherwise}, \end{cases}$$

where $x \in X$ and $y \in Y$. Find domain, range and height of R. Also calculate R^{-1}.

(2+3)+5

b) Find the relational join $P * Q$ of the following fuzzy relation and hence find composition $P \circ Q$: 6+4

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0.5 & 0.1 & 0 \\ 0.2 & 0 & 0.8 \\ 0 & 0.6 & 0.7 \\ 0.8 & 0 & 0.3 \end{bmatrix} \end{array} \text{ and } Q = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cc} \alpha & \beta \end{array} \\ \begin{bmatrix} 0.1 & 0.3 \\ 0.5 & 0 \\ 0.9 & 1 \end{bmatrix} \end{array}.$$

c) i) Solve the fuzzy equation: $A \bullet X = B$ where A, B are given by

$$A(x) = \begin{cases} x - 3 & \text{if } 3 \le x \le 4 \\ 5 - x & \text{if } 4 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

and $$B(x) = \begin{cases} \frac{x-12}{8} & \text{if } 12 \le x \le 20 \\ \frac{32-x}{12} & \text{if } 20 \le x \le 32 \\ 0 & \text{otherwise} \end{cases}.$$

ii) Show that a fuzzy set A on $\mathbb{R}$ is convex if and only if for all $x_1, x_2 \in \mathbb{R}$ and all $\lambda \in [0, 1]$,

$$A(\lambda x_1 + (1-\lambda) x_2) \ge minimum\{A(x_1), A(x_2)\}.$$

6+4

d) i) Establish the relationship between $\bigcup_{i \in I} {}^{\alpha}A_i$ and ${}^{\alpha}\left(\bigcup_{i \in I} A_i\right)$, where $A_i \in \mathcal{F}(X)$ for all $i \in I$, where I is an index set.

ii) Show by an example that if a fuzzy binary relation R is reflexive, then $R \subset R \circ R$.

6+4
