U.G. 6th Semester Examination - 2022

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-04A

(Mechanics)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) If the radial and transverse velocities of a particle are always proportional to each other, show that the path is an equiangular spiral.

b) A particle describes a circle under a force to a fixed point on the circumference. Find the law of force.

c) A point moves in a curve so that its tangential and normal accelerations are equal and the tangent rotates with constant angular vclocity. Obtain the path.

d) Explain the terms "apse" and "apsidal angle" related to a central orbit.

e) Explain the contexts where the terms "terminal velocity" and "escape velocity" appear respectively.

f) How does a rigid body differ from a deformable body?

g) Write down the conditions of equilibrium of a system of non-coplanar forces.

h) Find the moment of inertia of the perimeter of a circle about a tangent.

i) Is there any difference between "potential" and "potential energy"? Justify your answer.

j) When is the equilibrium of a heavy body resting on a fixed rough body said to be i) stable, ii) neutral?

k) A system of three particles move under their mutual attractions. How does their centre of mass move? Give reason.

l) State and explain the "energy test of stability" for a body resting on a fixed body.

m) Obtain the relation between the rate of change of angular momentum of a moving particle and force acting on it.

n) Write down the expressions of potential and kinetic energies of a simple pendulum of length l oscillating in a uniform gravitational field.

o) What is Poinsot's central axis? Write down its equations.

2. Answer any **four** questions: 5×4=20

a) Four forces, each of magnitude F act on a rigid body, three of forces act along the rectangular cartesian coordinate axes of x, y and z, while the fourth force acts along the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Find the equation of the central axis.

b) A uniform square of mass M is supported in a vertical plane on two smooth pegs at the same horizontal level. The distance between the pegs is l and the diagonal of the square is $d\,(<4l)$. If one diagonal is vertical and a mass m is attached to its lower end, prove that the equilibrium is stable if $4lm > M(d - 4l)$.

c) If the law of force be $2\mu\left(u^3-a^2u^5\right)$ and the particle be projected from an apse at a distance a with velocity' $\sqrt{\mu}/a$, show that it will be at a distance r from the centre after a time $\frac{1}{2\sqrt{\mu}}\left[r\sqrt{r^2-a^2}+a^2\cosh^{-1}(r/a)\right]$.

d) A uniform rod AB is held in a vertical position with the end A resting on a perfectly rough table and then released. Show that the end A does not leave the plane. Further, show that unless the plane is perfectly rough, the rod will begin to slip for some value of θ less than $\cos^{-1}(1/3)$, θ being the inclination of the rod to the vertical.

e) Prove that the angular momentum about a fixed point 0 of a rigid body of mass M moving in a plane is equal to $Mvp+MK^2\theta$ symbols have their usual meanings.

f) A particle falls from rest under gravity in a medium whose resistance is kv per unit mass, where v is the velocity of the particle and k is constant. Show that the distance traversed by the particle in time t is $\frac{g}{k^2}\left\{kt-1+e^{-kt}\right\}$.

3. Answer any **two** questions: 10×2=20

a) i) A particle describes a path which is nearly a circle about a centre of force $\mu\phi(u)$ per unit mass, u being the reciprocal of the distance from the centre of force and μ is a constant. Obtain the condition that this may be a stable motion.

ii) A particle of mass m moves in a central field of attractive force of which the intensity is $mkr^{-2}e^{-k^2}$, where k is a positive constant. Show that a circular orbit of radius r is stable if $r^2 > \frac{1}{2}$. 5+5

b) i) Two uniform similar rods of same material PQ and QT of length $2l$ and $2l'$ respectively are rigidly united at Q and suspended freely from P. If they rest inclined at an angle α and β respectively to the vertical, prove that $(l^2 + 2ll')\sin\alpha = l'^2 \sin\beta$.

ii) A heavy uniform rod of length $2a$ rests with its ends in contact with two smooth inclined planes of inclination α and β to the horizon. If θ be the inclination of the rod to the horizon, prove by the principle of virtual work that

$$\tan\theta = \frac{1}{2}(\cot\alpha - \cot\beta).$$ 5+5

c) i) Find whether a given straight line is at any point of its length a principal axis of a given material system and if so, obtain the other two principal axes.

ii) A uniform square lamina is bounded by the axes of x and y and the lines x = 2c, y = 2c and a corner is cut off by the line $\frac{x}{a} + \frac{y}{b} = 2$. Show that the principal axes at the centre of the square are inclined to the axes of x at angles is given by $\tan 2\theta = \frac{ab - 2(a+b)c + 3c^2}{(a-b)(a+b-2c)}$. 5+5

d) i) A heavy particle of mass m is projected from the lowest point of a smooth vertical circle and moves along the inner side of the circle. Discuss the motion of the particle.

ii) A parlicle is projected horizontally with velocity V along inside of a rough vertical circle from the lowest point. Provc that if it completes the circuit, it will return to the lowest point with a velocity v, given by

$$v^2 = V^2 e^{-4\pi\mu} - 2ag(2\mu^2 - 1)(1 - e^{-4\pi\mu})/(1 + 4\mu^2).$$ 5+5

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