U.G. 6th Semester Examination - 2022

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-04B

(Biomathematics)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

   a) What is non-hyperbolic equilibrium point of a dynamical system?

   b) Define asymptotic stability.

   c) State Hartman-Grobman theorem.

   d) What is basic reproduction number?

   e) State Routh-Hurwitz criterion.

   f) Define the Lyapunov stability of an equilibrium solution.

   g) Define phase-space in autonomous system.

   h) What is "Diffusive instability"?

   i) State Bendixson's negative criterion.

   j) Define autonomous and non-autonomous systems of differential equations.

   k) What is allee effect in biological systems?

   l) Define the terminology "Law of mass action".

   m) State the Poincare's theorem for critical points.

   n) Write the differential equations of growth of a microbial population on a single resource in a chemostat.

   o) What is activator-inhibitor system?

2. Answer any **four** questions: 5×4=20

   a) Discuss transcritical bifurcation of the system:

   $$\frac{dx}{dt} = \mu x - x^2,\ x \in R \text{ with } \mu \in R \text{ as a parameter.}$$

   b) Check whether the Hartman-Grobman theorem is applicable to the system

   $$\frac{dx}{dt} = rx - r_1 x^2 - cxy,\ \frac{dy}{dt} = cxy - dy$$

   of equilibrium point (0,0), where $r, r_1, c, d > 0$.

c) Describe Nicholson-Bailey model with all state variables and parameters. Find the equilibrium point of the model. 3+2

d) Find the phase path and draw it on the phase diagram

$$\frac{dx_1}{dt} = x_1 + x_2, \frac{dx_2}{dt} = x_1 - x_2 .$$

e) Deduce Hardy-Weinberg frequencies in genetics.

f) State the basic assumptions of classical Lotka-Volterra model for a predator-prey system.

3. Answer any **two** questions: 10×2=20

a) Discuss Gauss Competition model for two competing species. Find the equilibrium values and obtain the stability of the system about the equilibrium points. 3+4+3

b) Consider a simple model for a population with an exponential growth and simple Fickian equation

$$\frac{\partial n}{\partial t} = rn + D\frac{\partial^2 n}{\partial x^2}, n(0,t) = n(L,t) = 0; n(x,0) = n_0(x) .$$

Find the population size at any time t. Also find the critical length for which the population grow. (Where r and D are positive parameters). 8+2

c) Consider the epidemic model

$$\frac{dx}{dt} = -xy$$
$$\frac{dy}{dt} = (x-1)y$$
$$\frac{dz}{dt} = y$$

where x, y z are respectively the densities of susceptible, infected and removed classes at any time t. If x_0 is the number of initial susceptible, then prove that

i) There is no epidemic if $x_0 \leq 1$,

ii) There is an epidemic if $x_0 > 1$.

5+5

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