

U.G. 1st Semester Examination - 2021

MATHEMATICS

[HONOURS]

Generic Elective Course (GE)

Course Code : MATH-H-GE-T-01

(Algebra & Analytical Geometry)

Full Marks : 60

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**The notations and symbols have their usual meanings.*

1. Answer any **ten** questions: $2 \times 10 = 20$
- Find the modulus and principal value of the amplitude of $(\cos 50^\circ + i \sin 50^\circ)^6$.
 - Find all complex number z such that $\exp(z) = -i$.
 - Form an equation of fourth degree with integral coefficients having i and $\frac{1}{\sqrt{2}}$ as two of its roots.

[Turn over]

- If the sum of two roots of the equation $x^3 + px^2 + qx + r = 0$ is zero, prove that $pq = r$.
- Is the function $\frac{[x]}{[x]+1}$ one-one? Justify your answer.
- Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^3 + 2x - 3$. Find $f^{-1}(3)$.
- If a be an element of a multiplicative group with identity element e and if $a^2 = e$, show that $a = e$.
- If every element of a group be its own inverse then show that the group is abelian.
- Find the order of $[6]$ in the group \mathbb{Z}_{14} .
- Find the order of the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 6 & 5 & 4 & 2 \end{pmatrix}$$

- Transform the equation $y^2 - 2y = x$ with respect to parallel axes through $(-1, 1)$.
- Determine the nature of the conic $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0$.

m) Show that the equation

$$x^2 + 2\sqrt{3}xy + 3y^2 - 3x - 3\sqrt{3}y - 4 = 0$$

represents a pair of parallel straight lines.

n) Find the centre and radius of the circle

$$r = 3 \cos \theta + 4 \sin \theta.$$

o) Give an example of a relation on a set which is reflexive and symmetric but not transitive.

2. Answer any **four** questions: 5×4=20

a) Prove that $\sin\left(i \log \frac{a-ib}{a+ib}\right) = \frac{2ab}{a^2+b^2}$. 5

b) Solve the equation $x^3 - 24x + 27 = 0$ by Cardan's method. 5

c) Solve the following system of equations by Matrix method:

$$x + y + z = 2$$

$$2x - y + 3z = 1$$

$$3x + 2y - z = 1. \quad 5$$

d) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}. \quad 5$$

e) For any two elements a and b of a group G show that $(ab)^2 = a^2b^2$ if and only if $ab = ba$ in G . 5

f) A relation ρ is defined on the set \mathbb{Z} by ' $a\rho b$ ' if and only if $a-b$ is divisible by 5' for $a, b \in \mathbb{Z}$. Examine if ρ is an equivalence relation on \mathbb{Z} . 5

3. Answer any **two** questions: 10×2=20

a) i) Find the general and principal value of $(1+i)^{1-i}$. 6

ii) Apply Descartes' rule of signs to examine the nature of the roots of the equation $x^4 + 2x^2 + 3x - 1 = 0$. 4

b) i) On the set of integers \mathbb{Z} , the binary operation $*$ defined by $a*b = a+b-2$ for all $a, b \in \mathbb{Z}$. Show that $(\mathbb{Z}, *)$ is a group. 6

ii) Find A^2 and A^{-1} for the permutation

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}.$$

Is A^{-1} an even permutation? Justify your answer. 4

- c) i) Prove that the pair of the straight lines joining the origin to the points of intersection of the curves

$$ax^2 + 2hxy + by^2 + 2gx = 0 \text{ and}$$

$$a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$$

are perpendicular if

$$g'(a+b) = g(a'+b'). \quad 5$$

- ii) Show that the equation

$$4x^2 - 4xy + y^2 + 2x - 26y + 9 = 0$$

represents a parabola whose latus rectum

is $2\sqrt{5}$ units. 5
