U.G. 2nd Semester Examination - 2022

MATHEMATICS

[HONOURS]

Generic Elective Course (GE)

Course Code : MATH-H-GE-T-02

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Find the maximum value of $f(x) = 2 - |x|$.

b) If $f(t)$ be odd function of t then prove that $\int_a^x f(t)dt$ is even.

c) Examine that Rolle's theorem is applicable on the function $f(x)=|x-1|$ in (0,2).

d) If $f'(x)$ exist on [0,1], then show that $f(1) - f(0) = \frac{e-1}{e^x} f'(x)$.

e) Find the value of the limit $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$.

f) Applying MVT prove that $\frac{x}{1+x} < log(1+x) < x$, for all $x>0$.

g) Find $\int 5^{5^{5^x}} 5^{5^x} 5^x dx$.

h) If $u = sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, find value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

i) Find the singular solution of : $8ap^3 = 27y$.

j) Show that the differential equation $\left|\frac{dy}{dx}\right| + |y|=0, y(0)=1$ has no solution.

k) Find the degree and order of the differential equation $\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{2}{3}} = \frac{d^2y}{dx^2}$.

l) $I_n = \int_0^{\frac{\pi}{4}} tan^n x\, dx$, show that $I_n + I_{n-2} = \frac{1}{n-1}$.

m) State the Euler's theorem on homogeneous function.

n) Find the value of $\int_{-1}^{2}(|x| + [x])dx$.

o) Show that differential function is continuous.

2. Answer any **four** questions: 5×4=20

a) If $F(x,y)=0$, then show that $\frac{d^2y}{dx^2} = -\frac{(F_y)^2F_{xx}-2F_xF_yF_{xy}+(F_x)^2F_{yy}}{(F_y)^3}$.

b) If $u(x,y) = \phi(xy) + \sqrt{xy}\psi\left(\frac{y}{x}\right)$, $x \neq 0, y \neq 0$, where ϕ and ψ twice differentiable function, prove that $x^2\frac{\partial^2 u}{\partial x^2} - y^2\frac{\partial^2 u}{\partial y^2} = 0$.

c) If $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx$, n being positive integer greater than one, then show that $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$. Hence find the value of $\int_0^{\frac{\pi}{2}} x^5 \sin x \, dx$.

d) Solve by the method of variation of parameters:

$$\frac{d^3y}{dx^3} + \frac{dy}{dx} = cosec x.$$

e) State and prove the Cauchy mean value theorem.

f) Find general and singular solution of $y = px + \sin^{-1} p$.

g) Solve : $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 4y = xe^{-2x}$.

3. Answer any **two** questions: 2×10=20

a) i) If $y = \frac{\log x}{x}$, then prove that

$$y_n = \frac{(-1)^n n!}{x^{n+1}}\left[\log x - 1 - \frac{1}{2} - \frac{1}{3} - \ldots\ldots . \frac{1}{n}\right].$$ 5

ii) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that $u_{xx} + u_{yy} + u_{zz} = -\frac{1}{(x+y+z)^3}$. 5

b) i) Discuss the continuity of the function at the origin, $f(x) = \begin{cases} x \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. 4

ii) Find the Maclaurin series expansion of $\log(1+x)$ and find the region of validity of the expansion. 6

c) i) Show that the solution of $\frac{dy}{dx} + Py = Q$ can also be written in the form

$$y = \frac{Q}{P} - e^{-\int Pdx}\left[C + \int e^{\int Pdx} d\left(\frac{Q}{P}\right)\right].$$ 5

ii) Solve the differential equation by the method of undetermined coefficients:

$$(D^3 + 2D^2 - D - 2)y = e^x + x^2.$$ 5

d) i) Show that the equation $\frac{1}{(x-1)} + \frac{2}{(x-2)} + \frac{3}{(x-3)} = 0$ has roots in (1,2) and (2,3). 5

ii) If $y = \frac{x^3}{x^2 - 1}$, then prove that

$$(y_n)_0 = \begin{cases} 0, & \text{if } n \text{ is even} \\ -n!, & \text{if } n \text{ is odd} \end{cases}, n > 1.$$ 5

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