U.G. 3rd Semester Examination - 2020

MATHEMATICS

[PROGRAMME]

Course Code : MATH-G-CC-T-03

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

   a) Give an example of a countable (infinite) set with justification.

   b) Find Supremum and infimum of the set

   $$\left\{(-1)^n.\frac{n}{n+1}: n \in \boldsymbol{N}\right\}$$

   c) Using the completeness property of **R**, prove that the set **N** of all natural numbers is unbounded above.

   d) Show that finite subsets of real numbers have no limit points.

   e) Using Bolzano-Weierstrass theorem, show that of the $S = \left\{\frac{1}{n}: n \in \boldsymbol{N}\right\}$.

   f) Give an example of a sequence which is bounded, but not convergent.

   g) Show that every subsequence of a convergent sequence is convergent.

   h) Give an example of a monotonically increasing sequence which is not convergent. When does a monotonically increasing sequence converge?

   i) Does a series of positive terms neither convergent nor divergent? Give reason to your answer.

   j) Define p-series with its convergence criteria.

   k) Show that absolutely convergent series is convergent.

   l) Give an example of a conditional convergent series with justification.

   m) What is the difference between pointwise convergence and uniformly convergent sequence of functions?

   n) State M_n-test for a series of functions.

   o) Define the radius of convergence of power series.

2. Answer any **four** questions: 5×4=20

   a) State and prove Archimedean property of real numbers.

b) If A and B are closed sets, then prove that their union and intersection both closed.

c) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}}\right]=1.$$

d) What is a Cauchy sequence? Show that every convergent sequence of real numbers is a Cauchy sequence. Does the converse true?

e) If a series is convergent, then show that the sequence of the terms of the series converges to zero. Does the converse true?

f) Test the convergence of the series

$$\sum_{n=1}^{\infty}\frac{n^{n^2}}{(n+1)^{n^2}}$$

g) Using Abel's test show that the series

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\,x^n$$

is uniformly convergent on [0,1].

3. Answer any **two** questions: 10×2=20

a) State and prove Bolzano-Weierstrass theorem on the limit of point set. Give examples of (i) a closed set that is not open, (ii) a open set that is not closed, (iii) a set that is neither open nor

closed, (iv) an unbounded set with uncountable number of limit points, (v) a bounded set with only one limit point. 5+1+1+1+1+1

b) Prove that every sequence of real numbers has a monotonic subsequence. Prove that every convergent sequence is bounded, but not converse true. 5+(3+2)

c) Discuss the convergence of Geometric series. Define an alternating series. State for Leibnitz's test for alternating series. Show that the series

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$$

is convergent. 5+1+2+2

d) Using M_n-test, show that the sequence $\left\{\frac{x}{nx+1}\right\}$ converges uniformly to zero in [0,1]. State and prove Weierstrass' M-test for the convergence of series of functions. 5+5

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