U.G. 5th Semester Examination - 2021

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-2A

(Probability and Statistics)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 2×10=20

a) Find the expectation of a discrete random variable X whose probability density function is given by $f(x) = (1/4)^x$. $(x = 1,2,3 \ldots)$

b) A random variable X has the density function $f(x) = \frac{k}{1+x^2}$, where $-\infty < x < \infty$:

i) Find the value of the constant k

ii) Find the probability that X^2 lies between 1/3 and 1.

c) If $X^*=(X-\mu)/\sigma$ is a standardized random variable, prove that $(i)\ E(X^*) = 0,\ (ii)\ Var(X^*) = 1$.

d) Prove that $-1 \leq \rho \leq 1$, where ρ is the correlation coefficient.

e) Find the expectation of the sum of points in tossing a pair of fair dice.

f) Find i) the covariance, ii) correlation coefficient of two random variables X and Y if

$$E(X) = 2, E(Y) = 3, E(XY) = 10, E(X^2) = 9, E(Y^2) = 16.$$

g) State Chebyshev's inequality for a continuous random variable.

h) Find the characteristic function of a random variable X having density function $f(x) = Ae^{-\theta|x|}, -\infty < x < \infty$, where $\theta > 0$ and A is suitable constant.

i) Define coefficient of skewness and kurtosis of a distribution.

j) Find the probability that in successive tosses of a fair die, a 3 will come up for the first time on the fifth toss.

k) The joint density function of two continuous random variables X and Y is

$$f(x,y) = \begin{cases} \lambda xy & 0 < x < 4, 1 < y < 5 \\ 0 & otherwise \end{cases}$$

Find the value of the constant λ.

l) Define conditional expectation.

m) Show that $E[(X-c)^2]$ is minimum when $c = \mu = E(X)$.

n) Find the probability of not getting 7 or 10 in total on either of two tosses of a pair of fair dice.

o) Find the probability of drawing three cards at random from a deck of 52 ordinary cards if the cards are (i) replaced and (ii) not replaced.

2. Answer any **four** questions: 5×4=20

a) If X and Y are independent random variables, then show that

$$E(XY) = E(X)E(Y).$$ 5

b) Find the probability of getting a total of 11 (i) once, (ii) twice, in two tosses of a pair of fair dice. 5

c) Find the variance and standard deviation of the sum obtained by tossing a pair of fair dice. 3+2

d) Show that $E[(X-\mu)^2] = E(X^2) - [E(X)]^2$. Hence find $var(X)$ and σ_x, where $E(X) = 2, E(X^2) = 8$. 2+3

e) Show that if a binomial distribution with n =100 is symmetric; its coefficient of kurtosis is 2.9. 5

f) Define type-I and type-II errors.

The probability density function of the random variable X is $f(x) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & x > 0 \\ 0 & x \leq 0 \end{cases}$, where $\lambda > 0$. For testing the hypothesis $H_0: \boldsymbol{\lambda = 3}$ against $H_A: \boldsymbol{\lambda = 5}$ a test is given as "Reject X_0 if $X \geq 4.5$". Find the probability of type-I error and power of the test. 2+3

g) The standard deviation of the lifetimes of a sample of 200 electric bulbs was computed to be 100 hours. Find (i) 95%, (ii) 99% confidence limits for the standard deviation of all such electric light bulbs. 5

3. Answer any **two** questions: 10×2=20

a) A sample poll of 300 voters from district A and 200 voters from district B showed that 56% and 48%, respectively, were in favour of a given candidate. At a level of significance of 0.05, test the hypothesis that (i) there is a difference between the districts, (ii) the candidate is preferred in district A. (iii) Find the respective p values of the test. 4+3+3

b) Design a decision rule to test the hypothesis that a coin is fair if a sample of 64 tosses of the coin is taken and if a level of significance of (i)

0.05, (ii) 0.01 is used.

How could you design a decision rule to avoid a type-II error? 5+5

c) Find the probability that in 120 tosses of a fair coin (i) between 40% and 60% will be heads, (ii) $\frac{5}{8}$ or more will be heads. 5+5

d) Let X and Y be independent random variables having density function

$$f(t) = \begin{cases} 2e^{-2t} & t \geq 0 \\ 0 & otherwise \end{cases}.$$

Find $E(X+Y), E(X^2+Y^2)$ and $E(X, Y)$.

i) Does $E(X+Y) = E(X) + E(Y)$,

ii) $E(XY) = E(X)E(Y)$ *hold*?

Explain. 5+5
