CC 05 (Theory of Real Functions & Introduction to Metric Spaces)

F.M: 10 TIME: 30 MIN

* Required

1. Email *

2. NAME *

3. UNIVERSITY REGISTRATION NUMBER *

4. UNIVERSITY ROLL NUMBER *

Untitled Section

5.

1. A function f is defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} cos\frac{1}{x}\ for\ x \neq 0 \\ 0\ for\ x = 0 \end{cases}$$

Then

(i) f is continuous at x=0,

(ii) f is not continuous at x=0.

Mark only one oval.

- i
- ii

6.

2. A function f is defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}\ for\ x > 2 \\ 10\ for\ x = 2 \end{cases}$$

Then

(i) f has a removable discontinuity of first kind at x=2,

(ii) f has a non-removable discontinuity of first kind at x=2.

Mark only one oval.

- i
- ii

7.

3. A function f is defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} 1 \; if \; x \; is \; rational \\ 0 \; if \; x \; is \; irrational \end{cases}$$

Then

(i) $\lim_{x \to a} f(x)$ exists for each $a \in \mathbb{R}$

(ii) $\lim_{x \to a} f(x)$ does not exist for any $a \in \mathbb{R}$.

Mark only one oval.

- i
- ii

8.

4. A function f defined on (-1, 2) by

f(x)=|x|+|x-1|

Then

(i) $f'(x)$ does not exist at x=1,

(ii) $f'(x)$ exists at x=1 .

Mark only one oval.

- i
- ii

9.

5. A function f(x) defined on [a, a+h]. The Cauchy's form of remainder of its Taylor series expansion is

(i) $\frac{h^n}{n!} f^n(a + \theta h)$ for $0 < \theta < 1$,

(ii) $\frac{h^n(1-\theta)^{n-1}}{(n-1)!} f^n(a + \theta h)$ for $0 < \theta < 1$.

Mark only one oval.

- i
- ii

10.

6. $f(x) = x^5 - 5x^4 + 5x^3 + 12$ has extreme values at
   (i) 1,3 only
   (ii) x=0,1,3.

Mark only one oval.

- i
- ii

11.

7. $f(x) = x\sqrt{a^2 - x^2}$ is defined in $[0, a]$,
   (i) All the conditions of Rolle's theorems are satisfied,
   (ii) All the conditions of Rolle's theorems are not satisfied,

Mark only one oval.

- i
- ii

12.

8. A function $f(x) = x^2$, $x \in \mathbb{R}$,
   (i) is uniformly continuous in $[a, \infty)$ for $a \in \mathbb{R}, a \geq 0$,
   (ii) is not uniformly continuous in $[a, \infty)$ for $a \in \mathbb{R}, a \geq 0$.

Mark only one oval.

- i
- ii

13.

9. In a metric space (X, ρ), value of $\rho(x, x)$

(i) =0

(ii) >0

(iii) <0

(iv) None of above

Mark only one oval.

- i
- ii
- iii
- iv

14.

10. Space of continuous functions is a metric space

(i) True

(ii) False.

Mark only one oval.

- i
- ii