SECOND INTERNAL ASSESSMENT , 2019

DEPARTMENT OF MATHEMATICS

NABADWIP VIDYASAGAR COLLEGE

FULL MARKS: 40 TIME 1 H 30 MIN

CC-5

Answer any four questions: $5 \times 4 = 20$

5.1 Let f be a real valued continuous function in a closed interval [a,b]. Suppose that f(a) and f(b) are of opposite sign. Then prove that there exists at least one point c where a<c<b such that $f(c) = 0$.

5.2 Using $\varepsilon - \delta$ definition show that $\lim_{x\to 3} \frac{1}{x} = \frac{1}{3}$.

5.3 Verify Rolle's Theorem for $f(x) = 2x^3 + x^2 - 4x - 2$.

5.4 Show that $\frac{x}{1+x} < \log(1 + x) < x \ \ \forall x > 0$.

5.5 Expand $\sin x$ in power series expansion.

5.6 State Taylor's theorem with generalized form of remainder. Hence deduce Cauchy's form of remainder and Lagrange's form of remainder from this.

5.7 Prove that if (X,d) is a metric space and $x_1, x_2, x_3, \dots, x_n \in X$ then

$$d(x_1, x_n) \leq d(x_1, x_2) + d(x_2, x_3) + \cdots + d(x_{n-1}, x_n)$$

5.8 Prove that in a metric space arbitrary union of open sets is open.

CC-6

Answer any four questions: $5 \times 4 = 20$

6.1 Find the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 3 & 5 & 1 & 2 \end{pmatrix}$.

6.2 Examine if $(\mathbb{Z}, o)$ where $aob = a + b + ab, a, b \in \mathbb{Z}$ is a group or not.

6.3 Let (G, o) be a group and H,K are subgroups of (G, o). Prove that $H \cap K$ is a subgroup of (G, o).

6.4 Prove that the center of a group is a subgroup of that group.

6.5 Prove that every group of prime order is cyclic.

6.6 Let, G be a group and H be a subgroup of G. Let, $a, b \in G$. Then show that $aH = bH$ if and only if $a^{-1}b \in H$.

6.7 Prove that the intersection of two normal subgroup of a group G is normal in G.

6.8 Let (G, o) and $(G', *)$ be two groups and $\emptyset: G \to G'$ be a homomorphism. Then prove that $\emptyset(G)$ is a subgroup of G'.