Limit of a Function of Single Real Variable

Departmental Seminar on Mathematics and its Applications

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Definition

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1. x is a real variable
2. a is a finite real number- its value remain fixed
3. x passes successively through a number of values so that if we consider any particular value taken by x , then we may discriminate the values that precede and the values that follow and none of the values of x can looked upon as the last.

Definition

$x \rightarrow a$ or $\lim x = a$ means given any $\epsilon > 0$, the successive values of x will ultimately satisfy the inequality $0 < |x - a| < \epsilon$, i.e. ultimately x lies in the open interval $(a - \epsilon, a + \epsilon)$ $(x \neq a)$, i.e. ultimately x satisfies $a - \epsilon < x < a$ and $a < x < a + \epsilon$.

Definition

When $a - \epsilon < x < a$: We say that x tends to a from the left and we write $x \rightarrow a^{-}$ or $x \rightarrow a^{-0}$ but when $a < x < a + \epsilon$, we say that x tends to a from the right and we write $x \rightarrow a^{+}$ or $x \rightarrow a^{+0}$.

Definition

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When we write $x \longrightarrow +\infty$ or simply $x \longrightarrow \infty$, we assume x is a real variable which assumes successive values so that one can discriminate values that precede and the values that follow and that no value is the last value attained by x . ∞ is not a number at all, it is a symbol whose meaning will be made clear in the following lines.

Definition

An independent variable x tending to $-\infty$ written as $x \to -\infty$ is the same as $(-x) \to +\infty$.Hence $x \to -\infty$ means the successive values of x ultimately become and remain less than $-G$, where G is any arbitrary positive number , given in advance.

Limit of a Function:-

Meaning of $\lim_{x \to a} f(x) = l$ or $f(x) \to l$ as $x \to a$: For every positive number, $\exists$ a positive number δsuch that whenever $0 < |x - a| < \delta$,$|f(x) - l| < \epsilon$.

The definition requires that f must be defined in some deleted neighbourhood N of the point a.

The value of f at $x = a$, is left out of the discussion. We are just not concerned with $f(a)$.

Instead of $\lim_{x \to a} f(x) = l$, we may often write $f(x) \to l$ as $x \to a$.

Example

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We try to verify the validity of the statement $\lim_{x \to 2} 5x = 10$.The statement is true if , for any given $\epsilon > o$, $\exists \delta > 0$ s.t.$|5x - 10| < \epsilon$ whenever $0 < |x - 2| < \delta$.Now $|5x - 10| < \epsilon \Leftrightarrow |x - 2| < \epsilon/5$.Choosing $\delta = \epsilon/5$, our definition criterion is satisfied , hence the statement $\lim_{x \to 2} 5x = 10$ is true .

Example

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A very useful result: $\underset{x \to 0}{lim} x \sin(\frac{1}{x}) = 0$.This statement is true , if given any $\epsilon > 0$ we can find $\delta > 0$ s.t.

$$|x \sin(\frac{1}{x}) - 0| < \epsilon, \forall x \text{ in } 0 < |x - 0| < \delta.$$

But see that

$$|x \sin(\frac{1}{x}) - 0| = |x \sin(\frac{1}{x})| \leq |x|$$

[Since $|\sin(\frac{1}{x})| \leq 1$, except for $x = 0$, where it is undefined , in this case we are not concerned with the value of $x \sin(\frac{1}{x})$ at $x = 0$].Therefore, $|x \sin(\frac{1}{x}) - 0| < \epsilon$ is true if $|x| < \epsilon (x \neq 0)$ i.e , if $0 < |x - 0| < \delta$, where $\delta = \epsilon$.Thus given any $\epsilon > 0$, we can find δ $(= \epsilon)$ which will satisfy our definition test $\underset{x \to 0}{lim} x \sin(\frac{1}{x}) = 0$.

References

- Ghosh and Maity : An Introduction to Analysis-Differential Calculus.
- Shanti Narayan : Differential Calculus.
- Das and Mukherjee : Differential Calculus.
- S.C.Malik and S. Arora: Mathematical Analysis
- S.K.Mapa: Introduction to Real Analysis

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