NABADWIP VIDYASAGAR COLLEGE
PART III (HONS.), 2020
Paper – V

1. A string of length 'a' forms a shorter diagonal of a rhombus formed of four uniform rods, each of length 'b' and weight W, which is hinged together. If one of the rod is supported in a horizontal position, the tension of the string is

a) $\frac{W(b^2-a^2)}{4\sqrt{4b^2-a^2}}$ b) $\frac{2W(2b^2-a^2)}{4\sqrt{4b^2-a^2}}$ c) $\frac{W(b^2-2a^2)}{\sqrt{b^2-a^2}}$

2. Two equal forces act along the generators of the same system of the hyperboloid $\frac{x^2+y^2}{a^2}-\frac{z^2}{b^2}=1$ and cut the plane $z=0$ at the extremities of perpendicular diameter of the circle $x^2+y^2=a^2$, the pitch of the equivalent wrench is

a) $\frac{ab}{a^2+b^2}$ b) $\frac{ab^2}{2a^2+b^2}$ c) $\frac{a^2b}{a^2+2b^2}$

3. A thin hemispherical bowl of radius 'b' and weight 'w', rests in equilibrium on the highest point of a fixed sphere of radius 'a' which is rough enough to prevent any sliding. Inside the bowl is placed a small smooth sphere of weight 'W_1', the equilibrium is unstable unless

a) $W < W_1\left(\frac{a+b}{2b}\right)$ b) $W_1 < W\left(\frac{a-b}{2b}\right)$ c) $W_1 = W\left(\frac{a-b}{2a}\right)$

4. At the vertex C of a triangle ABC, which is right angled at C, the principal axis are inclined to sides at an angle

a) $\tan^{-1}\left(\frac{ab}{a^2-b^2}\right)$ b) $\tan^{-1}\left(\frac{2ab}{a^2-b^2}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{ab}{a^2-b^2}\right)$

5. If the time of complete oscillation of a compound pendulum is $2\pi\left(\frac{k^2}{gh}\right)^{\frac{1}{2}}$, then the length of simple equivalent pendulum is

a) $\frac{k^2}{g}$ b) $\frac{k^2}{h}$ c) $\frac{k^2}{gh}$

6. Suppose a rigid body is in motion. At any time 't' let 'r' be the position vector of mass 'm' of the body and F and R the external and internal forces respectively acting on it, then

a) $\sum F+\sum m\left(\frac{d^2r}{dt^2}\right)=0$ b) $\sum F+\left\{-m\left(\frac{d^2r}{dt^2}\right)\right\}=0$ c) $\sum R+\sum m\left(\frac{d^2r}{dt^2}\right)=0$

7. A semi-circular tube has its bounding diameter horizontal and contains equal volumes on 'n' fluids of densities successively equal to $\rho,2\rho,3\rho,\ldots$ arranges in this order, each fluid subtends an angle 2α at the centre and the tube just holds them all, then

a) $\tan 2n\alpha=(n+1)\tan\alpha$ b) $\tan 3n\alpha=(3n+2)\tan\alpha$

c) $\tan n\alpha = (2n+1)\tan\alpha$

8. If near the earth's surface gravity be assumed to be constant and the temperature in the atmosphere to be given by $t = t_0\left(1 - \frac{z}{nH}\right)$, where 'H' is the height of the homogeneous atmosphere, the pressure in the atmosphere will be given by the equation

a) $p_0 = p\left(1 - \frac{z}{nH}\right)^n$ b) $p = p_0\left(1 - \frac{z}{nH}\right)^n$ c) $p = 2p_0\left(1 - \frac{z}{2nH}\right)^{2n}$

9. A circular disc of radius 'a' is just completely immersed with its plane vertical in a homogeneous liquid, the distance between the centers of pressure of the two semicircles into which the disc is divided by its horizontal diameter is

a) $\frac{18\pi a}{9\pi^2-16}$ b) $\frac{16\pi a}{9\pi^2+16}$ c) $\frac{2\pi a}{4\pi^2+9}$

10. A solid paraboloid of sp. Gr. σ floats in a liquid of sp. Gr. ρ with its axis vertical and vertex downwards, the equilibrium will be stable if

a) $2a > h\left(1 - \sqrt{\frac{\rho}{\sigma}}\right)$ b) $3a > h\left(1 - \sqrt{\frac{\rho}{\sigma}}\right)$ c) $a > h\left(1 + \sqrt{\frac{\rho}{\sigma}}\right)$

Paper-VI

1. Which is the radius of convergence of the power series $\frac{1}{3} - x + \frac{x^2}{3^2} - x^3 + \frac{x^4}{3^4} - x^5 + \cdots$

(a) $\frac{1}{3}$ (b) -1 (c) 1

2. Let f(x) be defined on [a,b] by

f(x)=1 when x is rational,
=-1 when x is irrational.
Then,
(a) only f is integrable on [a,b] (b) only |f| is integrable on [a,b]
(c) both f and |f| are intrgrable on [a,b]

3. What is the value of $\int_0^\infty e^{-t}\, t^{\frac{5}{2}} dt$

(a) $\frac{15}{8}\sqrt{\pi}$ (b) $\frac{3}{4}\sqrt{\pi}$ (c) $\frac{3}{8}$

4. Use the relation $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$, find which is true

$(a)\frac{\sqrt{3}}{8} \le \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{sinx}{x} dx \le \frac{\sqrt{2}}{6}$, $(b) -\frac{\pi}{12} \le \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{sinx}{x} dx \le \frac{\pi}{12}$, $(c)\frac{\pi^2}{4} \le \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{sinx}{x} dx \le \frac{\pi^2}{3}$.

5. What is the maximum value of $f(x,y,z) = x^2y^2z^2$ subject to subsidiary condition $x^2 + y^2 + z^2 = c^2$ (x,y,z are positive)

(a) $\sqrt{3}\,c$ (b) $\frac{c^3}{3\sqrt{3}}$ (c)$\frac{c^6}{27}$

6. The value of $\iint_R xy(x^2+y^2)dxdy$ over $R:\{0 \le x \le a; 0 \le y \le a\}$is

$(a)\frac{a^2b^2}{12}(a^3+b^3)$ $(b)\frac{1}{8}a^2b^2(a^2+b^2)$ $(c)ab(a^2+b^2)$

7. The function f is defined by $f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2}, & where\ x^2+y^2 \neq 0 \\ 0, & where\ x^2+y^2=0 \end{cases}$

Then

$(a)f_{xy} = f_{xy}$ for every $(x,y) \in \mathbb{R}^2$ $(b)f_{xy} = f_{xy}$ only (0,0)

$(c)f_{xy} = f_{xy}$ for all $(x,y) \neq (0,0)$.

8. Let A and B be two subsets of the metric space (X,d), then which is not correct

$(a)A \subseteq B$implies $int\ A \subseteq int\ B$, $(b)int(A \cap B) = intA \cap intB$,
$(c)int(A \cup B) = intA \cup intB$.

9. Which is not true
(a) Any closed interval with usual metric space is compact,
(b) Every separable space is compact,
(c) Continuous image of a compact set is compact.

10. The Laplace transform of $F(t) = (t^2+1)^2$ is given by

$(a)\frac{1}{p^4+2p^2+1}$ $(b)\frac{24}{p^5}+\frac{4}{p^3}+\frac{1}{p}$ $(c)\frac{p^5}{5}+\frac{2p^3}{3}+p.$

Paper - VII

1. Vector Analysis was discovered by

a) Sir James Chadwick b) Sir Willard Gibbs c) Sir Anaximander

2. Find the divergence of the vector $F = (y-z)i + (z-x)j + (x-y)k$

a) 0 b) 1 c) 2

3. Two coins are tossed simultaneously. The probability (upto two decimal points accuracy) of getting at least one head is

a) 0.75 b) 0.50 c) 0.25

4. The probability that a number selected at random from the first 50 natural numbers is a composite number is

a) $\frac{17}{25}$ b) $\frac{8}{25}$ c) $\frac{8}{17}$

5. A simple of 15 data is as follows; 17, 18, 17, 17, 13, 18, 5, 5, 6, 7, 8, 9, 20, 17, 3. The mode of the data is

a) 15 b)17 c) 16

6. A six-face fair dice is rolled a large number of times. The mean value of the outcomes is

a) 4.5 b) 5.5 c)3.5

7. A coin is tossed thrice. Let XX be the event that head occurs in each of the first two tosses. Let YY be the event that a tail occurs on the third toss. Let ZZ be the event that two tails occur in three tosses. Based on the above information, which one of the following statements is TRUE?

a) X and Y are not independent b) Y and Z are dependent

c) Y and Z are independent

8. The chance of a student passing an exam is 20% The chance of a student passing the exam and getting above 90% marks in it is 5%. Given that a student passes the examination, the probability that the student gets above 90% marks is

a) $\frac{1}{18}$ b) $\frac{1}{4}$ c) $\frac{2}{9}$

9. Consider an anti – symmetric tensor P_{ij} with the indices i and j running from 1 to 5 . The number of independent component of the tensor is

a) 5 b) 25 c) 10

10. If z be a complex number then $\arg z + \arg \bar{z} =$

a) $2n\pi$ b) $n\pi$ c) $3n\pi$

Paper-VIII

1. If $f(x) = 2x^2 + 5x + 6$, then divided difference $f(3,1) =$

a) 15 b) 13 c) 11

2. In divided difference $f(x, x) =$

a) $f'(x)$ b) ∞ c) 0

3. $(11101)_2 + (101)_2 =$

a) $(110001)_2$ b) $(11111)_2$ c) $(100010)_2$

4. $(25)_{10}$ is equals to

a) $(11001)_2$ b) $(10011)_2$ c) $(11101)_2$

5. In 'C' language '%d' is used for

a) integer type b) long integer type c) fraction type

6. Value of $\int_0^1 4x\, dx$ taking 4 intervals by Trapezoidal Rule is

a) 2.5 b) 2 c) 3

7. In fixed point iteration method $x = \emptyset(x)$ will be converge in [a,b], If in [a,b]

a) $|\emptyset'(x)| = 0$ b) $|\emptyset'(x)| > 1$ c) $|\emptyset'(x)| < 1$

8. Rate of convergence of Newton Raphson method is

a) 2 b) $\frac{1}{2}$ c) 0

9. Newton forward difference formula is used for

a) equal intervals b) unequal intervals c) both

10. Relative Percentage error of $\frac{2}{3}$ is

a) 0.05 b) 0.005 c) 0.5