

TEST EXAMINATION: 2018

# MATHEMATICS (HONOURS)

VII+VIII PAPERS, FULL MARKS: 50+25, TIME: THREE HOURS

## Part-I

1. Answer all the questions.

$2 \times 5 = 10$

- State Serret-Frenet formula.
- Find the mean of  $X$ , whose probability density function is given by
$$f(x) = cx, 0 < x < 1$$
$$= 0, \text{ elsewhere}$$
- What do you mean by fixed point of a bilinear transformation?
- If  $a_j^i \lambda^j = 0$  for any arbitrary contravariant vector  $\lambda^j$ , show that  $a_j^i = 0$ .
- Give definition of unbiased estimate of parameter of a population.

2. Answer all the questions.

$(5 + 5) \times 4 = 40$

- a) (i) The joint probability density function of the random variables  $X$  and  $Y$  is

$$f(x, y) = K(1 - x - y), x \geq 0, y \geq 0, x + y \leq 1$$
$$= 0, \text{ elsewhere}$$

Where  $K$  is a constant. Prove that  $\text{cov}(X, Y) = \frac{-1}{80}$ .

(ii) Prove that  $u(x, y) = e^{-x}(x \sin y - y \cos y)$  is harmonic. Find  $v(x, y)$  such

that  $f(z) = u(x, y) + iv(x, y)$  is analytic.

- b) (i) Show that every  $(0,2)$  tensor can be expressed as the sum of two tensors of type  $(0,2)$ , one of which is symmetric and the other is skew symmetric.
- (ii) Verify Stoke's theorem for  $\vec{A} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$  where  $S$  is the upper half surface of the sphere  $x^2 + y^2 + z^2 = 1$  and  $C$  is its boundary.
- c) (i) If  $\vec{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$  evaluate  $\int_C \vec{A} \cdot d\vec{r}$  from the point  $(0,0,0)$  to  $(1,1,1)$ .

(ii) A die was thrown 60 times with the following results:

Face	1	2	3	4	5	6
Frequency	6	10	8	13	11	12

Are the data consistent with the hypothesis that the die is honest? Given that  $\chi_{0.01}^2 = 15.09$  for 5 degrees of freedom.

- d) (i) Find the confidence interval for population mean  $m$  for normal  $(m, \sigma)$  population, where  $\sigma$  is known.

(ii) The random variables X and Y are normally correlated with correlation coefficient  $\rho$ . Prove that  $\frac{X}{\sigma_x} + \frac{Y}{\sigma_y}$  and  $\frac{X}{\sigma_x} - \frac{Y}{\sigma_y}$  are independent normal variates.

### Part-II

**3. Answer all the questions.**

1 × 5 = 5

- i. State the composite form of Trapezoidal rule for numerical integration.
- ii. Define the term 'software'.
- iii. What do you mean by Algorithm?
- iv. Define relative error.  $\frac{E_a}{V_T}$
- v. Write the order of convergence of the regula falsi process for numerical solution of nonlinear equations.

**4. Answer all the questions.**

5 × 4 = 20

- a) Convert the decimal numbers 19 and 23 into their binary equivalents and obtain their product using binary arithmetic.
- b) Describe the Gauss-elimination method for a system of 3 equations with 3 unknowns.
- c) Establish Lagrange's Interpolation Formula.
- d) Write a program in C to find  $y(0.4)$  from the equation  $\frac{dy}{dx} = x' - y$ ,  $y(0) = 1$  taking  $h=0.1$  by Runge-Kutta method.