Internal Examination 2019
MATHEMATICS(HONOURS)
Seventh Paper

Full Marks-50 **Time- Two Hours**

Group-A

1.Answer any five question. $1 \times 5 = 5$

a) Define a mixed tensor of rank 2.

b) Define an irrotational field.

c) Given that $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(AB) = \frac{1}{4}$. State wheather the events A and B are independent or not.

d) Determine the argument of $1 - \cos x + i \sin x$.

e) State Gauss' divergence theorem.

f) Define a symmetric tensor.

g) Write down the probability density function for the Normal (m, σ) distribution.

h) Define an analytic function.

Group-B

2. Answer any Six question. $2 \times 6 = 12$

a) If $\emptyset(x, y, z) = x^2yz + 4xz^2$, find the directional derivative of $\emptyset$ at $(1, -2, -1)$ in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.

b) Find the circulation of $\vec{F}$ round the curve C, where $\vec{F} = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ and C is the circle $x^2 + y^2 = 1, z = 0$.

c) If the relation $b_j^i v_i = 0$ holds for an arbitrary covariant vector v_i , show that $b_j^i = 0$.

d) Show that an analytic function in a domain with its derivative zero for every point of the domain is constant.

e) Find the bilinear transformation which maps $1, -i, 2$ in the z-plane on $0,2, -i$ in the w-plane respectively.

f) Two cards are drawn successively from a pack without replacing the first. If the first card is a spade, find the probability that the second card is also a spade.

g) The random variable X is normal (m, σ). Find the distribution of $Y = aX + b$ where a, b are constants.

h) Show that the mean of the Cauchy Distribution does not exists.

i) Find the value of the constant k such that

$$f(x) = \begin{cases} kx(1-x) & 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

is a probability density function.

Group-C

3. Answer any three question. $6 \times 3 = 18$

a) state and prove Serret-Frenet formulae.

b) If $\left(\frac{x^1}{x^2}, \frac{x^2}{x^1}\right)$ is a covariant vector in Cartesian coordinates x^1, x^2 , find its components in polar coordinates r, θ.

c) Prove that the function $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic and find a function v such that $f(z) = u + iv$ is analytic.

d) The marks obtained by 17 candidates in an examination have a mean 57 and variance 64. Find 99% confidence limits for the mean of the population of marks, assuming it to be normal. Given $t_{0.01} = 2.961$ with 16 degrees of freedom.

e) Prove Schwartz's inequality for expectations that $[E(XY)]^2 \leq E(X^2)E(Y^2)$, and hence deduce that $-1 \leq \rho(X, Y) \leq 1$.

Group-D

4, Answer any three question. $15 \times 1 = 15$

a) i) Verify Stokes' theorem for the vector function $\vec{F} = (x^2 - y^2)\hat{\imath} + 2x\hat{\jmath}$ round the rectangle bounded by the straight lines $x = 0, x = a, y = 0, y = b$.

ii) Evalute the surface integral $\iint_S \vec{F} . \hat{n}\, dS$, where $\vec{F} = 6z\hat{\imath} - 4\hat{\jmath} + y\hat{k}$ and S is that part of the plane $2x + 6y + 3z = 10$ which is located in the first octant.

iii) State and prove the necessary condition for a complex function f(z) to be regular. 5+5+5

b) i) If X and Y are independent binomial variates (n_1, p) and (n_2, p) respectively, then prove that their sum $U = X + Y$ is a binomial $(n_1 + n_2, p)$ variate.

ii) If the regration lines are $x + 6y = 6$ and $3x + 2y = 10$, find the means and co-relation coefficient.

iii) Prove that the transformation of contravarient vectors form a group.

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