

Internal Examination 2022  
Semester VI  
CC 14 (Unit 1) 2<sup>nd</sup> Unit Test

*F.M.: 10*

*Time: 30 Min*

Answer any one questions.

$1 \times 10 = 10$

1. (i) Let  $V = R^3$  be a vector space over the field  $R$ . Find the dual basis of the basis  $\{(1,0,1), (1,2,1), (0,0,1)\}$ .

(ii) Let  $V = R^3$  and define  $f_1, f_2, f_3 \in V^*$  as follows:

$$f_1(x, y, z) = x - 2y, f_2(x, y, z) = x + y + z, f_3(x, y, z) = y - 3z$$

Prove that  $\{f_1, f_2, f_3\}$  is a basis for  $V^*$  and find a basis for  $V$  for which it is a dual basis.

5 + 5

2. Let  $f$  be a linear functional on  $R^2$  defined by  $f(x, y) = 2x + y$  and  $T: R^2 \rightarrow R^2$  be defined by  $T(x, y) = (3x + 2y, x)$ .

(i) Compute  $T^t(f)$ .

(ii) Compute  $[T^t]_{\beta^*}$ , where  $\beta$  is the standard ordered basis for  $R^2$  and  $\beta^* = \{f_1, f_2\}$  is the dual basis.

(iii) Compute  $[T]_{\beta}$  and  $([T]_{\beta})^t$  and compare your result with (ii)

3+5+2