

SECOND INTERNAL ASSESSMENT , 2019

DEPARTMENT OF MATHEMATICS

NABADWIP VIDYASAGAR COLLEGE

FULL MARKS: 40

TIME 1 H 30 MIN

CC-1

Answer any four questions:

$5 \times 4 = 20$

- 1.1 Find the constant a and b so that  $\lim_{x \rightarrow 0} \left( \frac{\sin 2x}{x^3} + \frac{a}{x^2} + b \right) = 1$
- 1.2 If the cost C of producing x unit of a particular commodity is  $C(x) = \frac{1}{8}x^2 + 5x + 98$  and the selling price P when x units are produced is  $P(x) = \frac{1}{2}(75 - x)$ , then determine the level of production that maximizes the profit.
- 1.3 Derive the reduction formula for  $\int (\log x)^n dx, n \geq 1$ . Hence evaluate  $\int (\log x)^4 dx$ .
- 1.4 Find the arc length of the curve  $y = \frac{x^3}{24} + \frac{2}{x}$  from  $x = 2$  to  $x = 3$
- 1.5 A sphere of radius r passes through the origin and meets the co-ordinate axes at P,Q,R. Prove that the triangle PQR lies on the sphere  $9(x^2 + y^2 + z^2) = 4r^2$ .
- 1.6 Find the equation of the cone whose vertex is at the point  $(\alpha, \beta, \gamma)$  and whose generating lines passes through the curve  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ .
- 1.7 Solve:  $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$ .
- 1.8 Solve:  $(x + \tan y)dy = \sin 2y dx$ .

CC-2

Answer any four questions:

$5 \times 4 = 20$

- 2.1 For n real numbers prove that A.M  $\geq$  G.M.
- 2.2 Solve the equation  $x^3 - 18x - 35 = 0$  by Cardan's Method.
- 2.3 Use Euclidean algorithm to find integers u and v satisfying  $30u + 72v = 12$ .
- 2.4 A relation  $\rho$  is defined on the set  $\mathbb{Z}$  by " $a\rho b$  if and only if  $a - b$  is divisible by 5" for  $a, b \in \mathbb{Z}$ . Examine if  $\rho$  is an equivalence relation on  $\mathbb{Z}$ .
- 2.5 Determine the condition for which the system

$$x + y + z = 1, \quad x + 2y - z = b, \quad 5x + 7y + az = b^2$$

has (i) only one solution (ii) no solution (iii) many solution.

- 2.6 Apply elementary row operation to reduce the matrix  $\begin{pmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{pmatrix}$  to a row echelon matrix.

- 2.7 Determine the rank of the matrix  $\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 3 & 6 & 6 & 3 \end{pmatrix}$ .

- 2.8 Find the eigen values and corresponding eigen vectors of the matrix  $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ .