Test Examination-2018

Sub: Mathematics (Honours)

Nabadwip Vidyasagar College

Paper: I **F.M.-100** **Time: 4 hours**

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1. Answer any five questions: $1\times 5=5$

a) Define commutative property of a binary operation.

b) State the Fundamental theorem of classical algebra.

c) Write the polar equation of the circle with radius r and centre at $\left(r,\frac{\pi}{2}\right)$.

d) Give an example of injective mapping.

e) Applying Descartes' rule of signs find the number of positive roots of the equation $x^4+2x^2+3x-1=0$.

f) What is the rank of unit matrix of order three? Justify your answer.

g) Does the equation $x^2+3xy+2y^2=0$ represents a pair of straight lines? If so, find the angle between them?

h) Show that the diagonals of a rhombus are at right angles.

2). Answer any ten questions: $2\times 10=20$

a) Find the principal value of $(1+i)^i$.

b) Give an example of mapping which is neither injective nor surjective.

c) What is the equivalence relation, give an example.

d) Prove that every orthogonal matrix is non-singular.

e) Find the values of c for which the plane $x+y+z=c$ touches the sphere $x^2+y^2+z^2-2x-2y-2z-6=0.$

f) Show that the three vectors $\bar{a}=2i-j+k,\ \bar{b}=i-3j-5k,\ \bar{c}=3i-4j-4k$ form the sides of a right-angled triangle.

g) Express the matrix $\begin{bmatrix}1&3&4\\7&2&6\\2&8&1\end{bmatrix}$ as a sum of a symmetric and skew-symmetric matrices.

h) Prove that the circles $x^2+y^2+2ax+c^2=0$ and $x^2+y^2+2by+c^2=0$ will touch each other if $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$.

i) For what values of μ, the equation $xy+5x+\mu y+15=0$ represents a pair of straight lines?

j) If $|\vec{\alpha}+\vec{\beta}|=|\vec{\alpha}-\vec{\beta}|$, then show that $\vec{\alpha}$ is perpendicular to $\vec{\beta}$.

k) In a group $(G,\circ)$, b is an element of order 30. Find the order of b^{18}.

l) If α,β,γ are the roots of the equation $ax^3+bx^2+cx+d=0,(a\neq 0)$, find the value of $\sum\beta^2$.

3). <u>Answer any five questions</u>: $5\times 15=75$

a) (i)Solve the following equation by Cardan's method

$$x^3+3x+1=0.$$

(ii) If α,β,γ be the roots of the equation $x^3-px^2+qx-r=0,$ then form the equation whose roots are

$$\beta\gamma+\frac{1}{\alpha},\gamma\alpha+\frac{1}{\beta},\alpha\beta+\frac{1}{\gamma}.$$

(iii)Define adjoint of a square matrix. Show that for a square matrix of order n,

$$A(adjA)=(adjA)A=(\det A)I_n$$

(5+5+5)

b) (i)Show that a skew-symmetric determinant of order 3 is zero.

(ii)State and prove Lagrange's theorem of groups.

(iii)Find the rank of the matrix $\begin{pmatrix}1&2&3\\2&1&0\\0&1&2\end{pmatrix}$.

(iv) Write down the matrix

$$\begin{bmatrix}-2&3&2\\6&0&3\\4&1&-1\end{bmatrix}$$

as the product of elementary matrices. (4+5+2+4)

c) (i) If the St. lines $ax^2-2hxy+by^2=0$ form an equilateral triangle with the St. line $x\cos\alpha+y\sin\alpha=p$, show that $\dfrac{a}{1-a\cos 2\alpha}=\dfrac{h}{2\sin 2\alpha}=\dfrac{b}{1+2\cos 2\alpha}$.

(ii) If $a^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of St. lines, show that the area of the triangle formed by the bisectors of the angles between them and the axis of x is

$$\frac{\sqrt{(a-b)^2+4x^2}}{2x}\cdot\frac{ca-g^2}{ab-h^2}.$$

(iii) If α,β,γ be unit vectors satisfying the condition $\alpha+\beta+\gamma=0,$ then show that

$$\alpha.\beta+\beta.\gamma+\gamma.\alpha=-\frac{3}{2}.$$

(5+5+5)

d) (i)Find the equation of the plane passes through the point (2,1,-1) and is orthogonal to each of the planes $x-y+z=1$ and $3x+4y-2z=0.$

(ii) Show that the equations of the planes passing through the points (0,4,-3), (6,-4,3) and cutting of intercepts from the axes whose sum is zero, are $2x-3y-6z=6$ and $6x+3y-2z=18$.

(iii) Show that the volume of the parallelepiped whose edge are represented by $3\hat{i}+2\hat{j}-4\hat{k}$, $3\hat{i}+\hat{j}+3\hat{k}$ and $\hat{i}-2\hat{j}+\hat{k}$ is 49cubic units. (5+5+5)

e)(i) Find the image of the point (-3,8,4) in the plane $6x-3y-2z+1=0$.

(ii) Find the centre and radius of the circle $x^2+y^2+z^2=25, x+2y+2z+9=0$.

(iii) If $f: A \to B$ and $g: B \to C$ be both bijective, show that the composite mapping $g \circ f: A \to C$ is bijective. What conclusion can you draw about its converse ? (5+5+5)

f)(i) Show that the volume of the tetrahedron formed by points (0,0,0), (3,2,-1), (4,1,4) and (5,3,5) is $\frac{14}{3}$ cubic units.

(ii) Find the coordinates of the limiting points of the circles $x^2+y^2+2x+4y+7=0$ and $x^2+y^2+4x+2y+5=0$.

(iii) In a group $(G,*)$ the elements a and b commute and $O(a)$ and $O(b)$ are prime to each other. Show that $O(a*b)=O(a).O(b)$. Here $O(a)$ means the order of the element a.

(5+5+5)

g) (i) Prove that the straight lines joining the origin to the points of intersection of the straight line $\frac{x}{\alpha}+\frac{y}{\beta}=2$ and the circle $(x-\alpha)^2+(y-\beta)^2=\gamma^2$ are at right angles if $\alpha^2+\beta^2=\gamma^2$.

(ii) Find the eccentricity, latus rectum and the directrix of the following conics

$$\frac{4}{r}=3-5\cos\left(\theta-\frac{\pi}{4}\right).$$

(iii) Prove that any two left cosets of a subgroup H have the same cardinality. (5+5+5)