Test Examination-2018
Sub: Mathematics (Honours)
Nabadwip Vidyasagar College

Paper: III **F.M.-50** **Time: 2 hours**

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(1) Answer any **four** questions: 1 x 4 = 4

a) State Archimedean property of real numbers.

b) Show that $\emptyset(5040) = 1152$.

c) Define the Division ring.

d) Show that if a/b and b/c, then a/c.

e) Find the eigen values of the matrix $\begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix}$.

(2) Answer any **four** questions: 2 x 4 = 08

a) Using Dedikind Cut, define the real number $2\sqrt{2}$.

b) State Peano's axioms on natural number.

c) Find the infimum and the supremum of the following set : $\left\{1 + \frac{1}{\mathrm{n}} : \mathrm{n} \in \mathrm{N}\right\} \cup \left\{-1 - \frac{1}{\mathrm{n}} : \mathrm{n} \in \mathrm{N}\right\}$.

d) State Riemann's rearrangement theorem.

e) State Cantor-Dedekind axiom.

(3) Answer any **three** questions: 6 x 3 = 18

a) i) If n be a positive integer >1, prove that $n(n+1)^2 > 4(n!)^{\frac{3}{2}}$.

ii) Using principle of induction, prove that $3^{2n} - 8n - 1$ is divisible by 64. (3+3)

b) i) Test whether the following set of vectors (1, 2, -1), (3, -1, 2) and (5, 3,0) in Euclidean 3-space is linearly dependent or linearly independent.

ii) Prove that a finite integral domain is a field. (3+3)

c) i) Examine the validity of the hypothesis and the conclusion of Rolle's theorem for the function $\mathrm{f(x)} = \mathrm{x}^3 - 4\mathrm{x}$ in [-2,2].

ii) Use Taylor's theorem show that $\mathrm{x} - \frac{\mathrm{x}^3}{6} < sinx < x$ for x>0. (3+3)

d) i) Define piecewise continuous function. Give example with illustration of a function which is continuous but not uniformly continuous. (2+4)

(4) Answer any **two** questions: 10 x 2 = 20

a) i) Use Gram Schmidt process to obtain an orthogonal basis from the basis set {(1, 0, 1), (1, 1, 1), (1, 3, 4) of the Euclidean space $\mathbb{R}^3$ with standard inner product.

ii) Let f and g be two permutations defined by $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$ find $f^{-1}g^{-1}$. (5+5)

b) i) Define Cauchy Sequence. Show that $\{n^2\}$ is not a Cauchy sequence. Using Cauchy's second limit theorem prove that $\lim_{n\to\infty} \left(\frac{2n!}{n!n!}\right)^{\frac{1}{n}} = 4$.

ii)State and prove the Sandwich theorem. (1+2+3+4)

c) i) Show that $\left\{\frac{1}{n^p}\right\}$ is a null sequence for p>0. Prove that the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$ is convergent.

iii) State and prove Euclid's first theorem. (3+4+3)