Test Examination-2018

Nabadwip Vidyasagar College

Sub: Mathematics (Honours)

Paper: IV **F.M.-50** **Time: 2 hours**

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1. **Answer any two questions**: 6x2=12

a) Using two phase method solve :

$\text{Max } Z = 2x_1 + x_2 - x_3$

subject to $4x_1 + 6x_2 + 3x_3 \leq 8$

$3x_1 - 6x_2 - 4x_3 \leq 1$

$2x_1 - 3x_2 - 5x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$

b) Solve the L.P.P.

Minimize $Z = 2x_1 - 3x_2$

subject to $-x_1 + 2x_2 \leq 4$

$x_1 + x_2 \leq 6$

$x_1 + 3x_2 \geq 9$

$x_1, x_2 \geq 0$

c) Solve the following travelling salesman problem:

	A	B	C	D	E
A	–	14	10	24	41
B	6	–	10	12	10
C	7	13	–	8	15
D	11	14	30	–	17
E	6	8	12	16	–

d) Solve the following transportation problem:

	D_1	D_2	D_3	a_i
O_1	8	7	3	60
O_2	3	8	9	70
O_3	11	3	5	80
b_i	50	80	80	

2. **Answer any one question**: 8x1=8

a) i. Give an example of convex polyhedron.

ii. Examine whether the set is convex or not

$$S = \{(x_1, x_2) : x_1 \geq 2, x_2 \leq 3\}.$$

iii. $x_1 = 2, x_2 = 4$ and $x_3 = 1$ is a feasible solution to the system of equations

$$2x_1 - x_2 + 2x_3 = 2$$
$$x_1 + 4x_2 = 18$$

Reduce the feasible solution to a basic feasible one. (1+2+5)

b) i. What do you mean by 'Saddle Point' in a game.

ii. Find the dual of the following problem:

Minimize $Z = 3x_1 - 2x_2$

Subject to $2x_1 + x_2 \leq 1.$

$-x_1 + 3x_2 \geq 4.$

$x_1, x_2 \geq 0.$

iii. Solve the following game problem

	B_1	B_2	B_3
A_1	1	-1	3
A_2	3	5	-3
A_3	6	2	-2

(1+2+5)

3. **Answer any three questions**: 6x3=18

a) A particle is projected upwards with a velocity U in a medium whose resistance varies as the square of the velocity. Prove that it will return to the point of projection with velocity $v = \frac{UV}{\sqrt{U^2+V^2}}$ after a time $\frac{V}{g}\left\{\tan^{-1}\frac{U}{V} + \tan^{-1}\frac{v}{V}\right\}$, where V is the terminal velocity.

b) A heavy particle slides down a rough cycloid of which the coefficient of friction is μ. Its base is horizontal and vertex downwards. If it starts from a cusp and comes to rest at the vertex, then prove that $\mu^2 e^{\mu\pi} = 1$.

c) A particle describes a plane curve under the action of a central force ρ per unit mass. Prove that in usual notation the differential equation of the path of the particle is $\frac{h^2}{p^3}\frac{dp}{dr} = \rho$.

d) State the principle of the conservation of linear momentum. Show that change in kinetic energy of a particle is equal to the work done by external forces.

e) Find the tangential and normal components of acceleration of a particle moving along a plane curve.

4. **Answer any one question:** 7x1=7

a) i. Explain what are meant by 'impulse of a force' and 'impulsive force'.

ii. A particle moves with a central acceleration $u\left(r + \frac{a^4}{r^3}\right)$ being projected from an apse at a distance a with a velocity $2\sqrt{u}a$. Prove that it describes the curve $r^2\left(2 + cos\sqrt{3}\theta\right) = 3a^2$.

(2+5)

b) i. State Kepler's laws of planetary motion.

ii. A gun of mass M fires a shell of mass m horizontally and energy of explosion is such as would be sufficient to project the shell vertically to the height h. Show that velocity of the recoil of the gun is $\left(\frac{2m^2gh}{M(m+M)}\right)^{\frac{1}{2}}$. (2+5)

5. **Answer any one question:** 5x1=5

a) i. Find the stationary points of $f(x) = x^2 + 4x + 2$.

ii. Evaluate $\lim_{n\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$. (2+3)

b) i. Show that $f(x) = x^3 - 6x^2 + 24x + 14$ has neither maximum or minimum.

ii. Find the maximum and minimum value of $x + \sin 2x$ for $0 < x < \pi$. (2+3)