DEPARTMENT OF MATHEMATICS

TEST EXAM 2019, PAPERS III

FM-50, TIME-2HOUR

GROUP-A

Answer any five questions. $5 \times 5 = 25$

1) If a, b, c, d be positive numbers, not all equal, prove that

$$(a = b + c + d)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right) > 16$$

2) State and prove division algorithm.

3) Prove that the linear sum of two subspaces U and W of a vector space V over a field f is the smallest subspace containing U and W.

4) Prove that the set $S = \{(2,2,1), (1,2,1), (1,1,2)\}$ is a basis of $\mathbb{R}^3$.

5) Prove that the eigen values of a real symmetric matrix are all real.

6) Prove that a subgroup H of a group G is normal if and only if $aHa^{-1} = H$ for every a in G.

7) Let $\emptyset: (G, o) \to (G', *)$ be a homomorphism. Then prove that $\emptyset$ is one-to-one if and only if $ker\emptyset = \{e_G\}$.

GROUP-B

Answer any five questions. $5 \times 5 = 25$

8) Show that between any two real numbers a and b $(a \neq b)$ there exists a rational number.

9) Show that the set of all irrational number is neither open nor closed.

10) Show that the derived set of a set is closed.

11) Show that every monotonic increasing sequence which is bounded above is convergent.

12) State and prove Bolzano-Weierstrass Theorem.

13) State and prove Cauchy's first limit theorem.

14) Show that the series $\sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n}$ is convergent.

DEPARTMENT OF MATHEMATICS

TEST EXAM 2019, PAPERS IV

FM-50, TIME-2HOUR

GROUP-A: Answer any five questions: $5 \times 5 = 25$

1. Use duality to solve the problem

$$\begin{aligned} Max\ Z &= 3x_1 - 2x_2 \\ subject\ to\ x_1 &\leq 4 \\ x_2 &\leq 6 \\ x_1 - x_2 &\leq 5 \\ x_2 &\geq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2. Solve the following Travelling Salesman problem:

	A	B	C	D	E
A	∞	2	4	7	1
B	5	∞	2	8	2
C	7	6	∞	4	6
D	10	3	5	∞	4
E	1	2	2	8	∞

3. Using two phase method solve

$$\begin{aligned} Max\ Z &= 2x_1 + x_2 - x_3 \\ subject\ to\ 4x_1 + 6x_2 + 3x_3 &\leq 8 \\ 3x_1 - 6x_2 - 4x_3 &\leq 1 \\ 2x_1 - 3x_2 - 5x_3 &\geq 4 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

4. In a rectangular game, the pay off matrix is given below

$$\begin{bmatrix} 2 & 2 & 1 & -2 & -3 \\ 4 & 3 & 4 & -2 & 0 \\ 5 & 1 & 2 & 5 & 6 \end{bmatrix}$$

Use dominance to reduce the game into 2×2 game and solve it.

5. $x_1 = 1, x_2 = 3, x_3 = 2$ is a feasible solution of the equations

$$2x_1 + 4x_2 - 2x_3 = 10$$
$$10x_1 + 3x_2 + 7x_3 = 33$$

Reduce the above feasible solution to a basic feasible solution.

6. Solve the following transtransportation problem:

	D_1	D_2	D_3	D_4	a_i
O_1	2	5	4	7	4
O_2	6	1	2	5	5
O_3	4	5	2	4	4
b_j	3	7	6	2	

GROUP-B: Answer any five questions: $5 \times 5 = 25$

1) A particle starts from rest and the acceleration at any time t is $(f - kt^2)$, where f and k are positive constants. Show that the maximum velocity u attained by the particle is given by $u = {}^2/_3\sqrt{\frac{f^3}{k}}$ and it moves over the space $\frac{15u^2}{16f}$ before it attains this maximum velocity.
2) A particle moves in a straight line . Its acceleration is directed towards a fixed point O in the line and is always equal to $\mu(\frac{a^5}{x^2})^{1/3}$, when it is at a distance x from O. If it starts from rest at a distance a from O, then show that it will arrive at O with velocity $a\sqrt{6\mu}$ after time ${}^8/_{15}\sqrt{\frac{6}{\mu}}$.
3) A particle of mass m moves in a straight line under an attractive force mn^2x towards a fixed point on the line when at a distance x from it. If it be projected with a velocity V towards the centre of force from an initial distance a from it, then prove that it reaches the centre of force after a time $\frac{1}{n}\tan^{-1}\frac{na}{V}$.
4) A particle is oscillating in a straight line about a centre of force O. When the particle is ata distance x from O, its acceleration is n^2x directed towards O and the amplitude of acceleration is a. When at a distance ${}^1/_2\, a\sqrt{3}$ from O, the particle receives a blow in the direction of motion which generates a velocity na. If the velocity be away from O, then show that the new amplitude is $\sqrt{3}$.
5) A particle is describing a circle of radius a in such a way that its tangential acceleration is k times the normal acceleration, where k is a constant. It the speed of the particle at any point be u, then prove that it will return to the same point after a time $\frac{a}{ku}(1 - e^{-2\pi k})$.
6) A particle falls from rest under gravity, from a height h in a resisting medium, whose resistance varies as the square of the velocity. Show that the particle does not acquire the terminal velocity during its motion and show that the time to reach the ground is $\frac{c}{g}\cosh^{-1}\left(e^{\frac{gh}{c^2}}\right)$, c being the terminal velocity.