NABADWIP VIDYASAGAR COLLEGE
TEST EXAMINATION 2020
PAPER:-V

FULL MARKS: 50 **TIME: 2 HOURS**

1. Answer any five questions: $\mathbf{6 \times 5 = 30}$

a) A uniform rod OA of length 2a, free to turn about its end O, revolves with uniform angular velocity $'2\omega'$ about the vertical OZ through O and is inclined at a constant angle α to OZ. Show that the value of α is either zero or $\cos^{-1}\left(\frac{3g}{16a\omega^2}\right)$.

b) A rigid body is moving under two dimension. Prove that the kinetic energy of the rigid body is $\frac{1}{2}MV^2 + \frac{1}{2}MK^2\dot{\theta}^2$. Notations are usual.

c) Show that three coplanar forces P,Q,R acting at the points A,B,C are in astatic equilibrium if they meet at a point on the circumcircle of the triangle ABC and if $P: Q: R = a: b: c$, Where are the sides of the triangle ABC.

d) A sphere of weight W and radius r lies within a fixed spherical shell of radius R and a particle of weight w is attached to its highest point, show that the equilibrium is stable if $W > \frac{R-2r}{r}w$.

e) Define angular momentum of a rigid body about a line. State and prove the principle of conservation of angular momentum of a rigid body moving under finite and impulsive forces.

f) A quadrant of a circle is immersed in a liquid with a bounding radius in the surface; find the position of the centre of pressure.

g) Show that the equation of the momental ellipsoid at the centre of a cube of side 2a referred to its principle axes is $2x^2 + 11(y^2 + z^2) = constant$.

h) If near the earth's surface, gravity be assumed to be constant and the temperature in the atmosphere is given by $t = t_0\left(1 - \frac{z}{nH}\right)$, where H is the height of the homogeneous atmosphere. Show that the pressure in the atmosphere will be $p = p_0\left(1 - \frac{z}{nH}\right)^n$.

2. Answer any two questions: $\mathbf{10 \times 2 = 20}$

a) State and establish the principle of virtual work for a system of co-planer forces acting on a rigid body.

b) Prove that the necessary condition for equilibrium of a perfect fluid under a system of forces whose components per unit mass at the point (x, y, z) parallel to the rectangular axes are X,Y,Z is
$X\left(\frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}\right) + Y\left(\frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z}\right) + Z\left(\frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x}\right) = 0.$

c) An area is bounded by two concentric semicircles with common bounding diameter in the surface; Show that the depth of its centre of pressure is $\frac{3\pi}{16}.\frac{(a+b)(a^2+b^2)}{a^2+ab+b^2}$.

d) Five equal uniform rods, each of weight w are freely joined together to form a pentagon ABCDE, which is suspended from the joint A and maintained in the shape of a regular pentagon by two strings joining A to C and D. Show that the tension of either strings is $2w\cos 18^\circ$.