NABADWIP VIDYASAGAR COLLEGE
TEST EXAMINATION 2020
PAPER:-VI

FULL MARKS: 50 **TIME: 2 HOURS**

1. Answer any eight questions: $\mathbf{2 \times 8 = 16}$

a) Give an example of a map $f: R^2 \to R$ such that $f_x(0,0)$ exists while $f_y(0,0)$ does not exists.

b) Let $f: [0,1] \to [0,1]$ be defined by $f(x) = \begin{cases} 1 \; if \; x \; is \; rational \\ -1 \; if \; x \; is \; irrational \end{cases}$. Compute upper integral and lower integral of f.

c) If $f_n(x) = x^n \forall x \in [0,1]$ then show that $\{f_n\}_{n=1}^{\infty}$ is not uniformly convergent on [0,1].

d) Find radius of convergence of the power series $\sum \frac{x^n}{n^2}$.

e) Evaluate $\int_0^1 \int_0^2 x^3 y \, dxdy$.

f) Let $f: R^2 \to R$ be defined by $f(x,y) = xy \; \forall (x,y) \in R^2$. Show that f is differentiable everywhere.

g) Prove that every discrete metric space is complete.

h) Prove that in a metric space complement of every singleton set is open set.

i) Verify implicit function theorem for the function $f(x,y) = x^2 + y^2 - 1$ at (0,1).

j) Show that in a metric space each open ball is an open set.

k) For any three points x, y, z in a metric space (X, d), show that $|d(x,z) - d(y,z)| \le d(x,y)$.

l) Using $\varepsilon - \delta$ definition show that

$$\lim_{(x,y)\to(0,0)} \left(x \sin\frac{1}{y} + y \sin\frac{1}{x} \right) = 0$$

2. Answer any four questions: $\mathbf{6 \times 4 = 24}$

a) Define oscillatory sum of a function $f(x)$ for a partition P in [a,b]. State and prove the necessary and sufficient condition for a function to be R-integrable in terms of oscillatory sum. $1 + 1 + 4$

b) State Dirichlet's condition. Expand in Fourier series $x^2 + x$ on $-\pi < x < \pi$ and deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ $2 + 3 + 1$

c) Let $g: R^2 \to R$ be defined by

$$g(x,y) = \begin{cases} \dfrac{xy}{x^2 + y^2}, for \; (x,y) \neq (0,0) \\ 0 \quad , for \; (x,y) = (0,0) \end{cases}$$

Prove that g is continuous everywhere except (0,0), even though the partial derivatives exists at (0,0). Check whether the function is differentiable at (0,0). $4 + 2$

d) Find the stationary points of the function $f(x,y,z) = x + y + z$ on the surface $S = \{(x,y,z) \in R^3 : ax^2 + by^2 + cz^2 = 1\}$. Determine maximum and minimum values of f on the surface S. 3+3

e) Solve $x^2 p + y^2 q = (x + y)z$

f) Using laplace transform solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{-x}$, given $y(0) = y'(0) = 0$.

g) Establish Cantor's Intersection Theorem in a complete metric space.

3. Answer any one question: $\mathbf{10 \times 1 = 10}$

a) i) Show that the integral $\int_0^{\frac{\pi}{2}} \log \sin x \; dx$ is convergent and hence evaluate it.

ii) Prove that a compact subset of a metric space $(X.d)$ is closed and bounded. $5 + 5$

b) i) Solve $(1 - t^2)\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = 0$ in power of t.

ii) State and prove sufficient condition for differentiability of a function of two variables. $5 + 5$