Test Examination 2019

Nabadwip Vidyasagar College

Sub: Mathematics (Honours) **Paper: VI** **F.M.-50** **Time: 2 hours**

(1) Answer any **five** questions: 1 x 5 = 5

a) Define radius of convergence of a power series $\sum a_n x^n$.
b) Give an example of a totally bounded metric space.
c) State Bonnet's form of the Second Mean Value Theorem.
d) State Young's Theorem.
e) Show that $\frac{9}{10}\Gamma\left(\frac{8}{3}\right) = \Gamma\left(\frac{2}{3}\right)$
f) If $a > 0$ then prove that $\int_a^\infty \frac{1}{x} dx$ does not exist.
g) When a map $f: \mathbb{R}^2 \to \mathbb{R}$ is said to be differential at a point (a, b).

(2) Answer any **five** questions: 2 x 5 = 10

a) Define stationary point and saddle point of a function.
b) Prove that an infinite discrete metric space cannot be totally bounded.
c) Find the radius of convergence of the power series $\sum \frac{x^n}{n^2}$.
d) Let (X, d) be a metric space. Show that the (X, d_1) is also a metric space, where $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$.
e) Using Weierstrass' M-Test, show that the series $\sum_{n=1}^{\infty} \frac{\cos nx}{n^p}$ (p>1) is uniformly convergent on $(-\infty, \infty)$.
f) Verify implicit function theorem for the function $f(x, y) = x^2 + y^2 - 1$ at (0,1).
g) Let f: $[0,1] \to [0,1]$ be defined by

f(x)=1, x is rational

=-1, x is irrational

Compute upper integral and lower integral of f.

(3) Answer any **three** questions: 5 x 3 =15

a) Solve $x^2 p + y^2 q = (x + y)z$.
b) Show that $\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$.
c) Find the Jacobian $\frac{\partial(y_1, y_2)}{\partial(x_1, x_2)}$, where $y_1 = x_1(1 - x_2)$ and $y_2 = x_1 x_2$.
d) Expand $f(x, y) = x^2 y + 3y - 2$ in power of (x-1) and (y-2).
e) If f: $[a, b] \to \mathbb{R}$ is monotonic, then prove that f is Riemann integrable.

(4) Answer any **two** questions: 10 x 2 = 20

a) i) ii) Show that the function f, where

$$f(x, y) = \frac{xy}{\sqrt{x^2+y^2}} \text{ if } x^2 + y^2 \neq 0$$
$$= 0, \text{ if } x = y = 0$$

is continuous, possesses partial derivatives but is not differentiable at the origin.

iii) Let $\pi: \mathbb{R}^2 \to \mathbb{R}$ be defined by $\pi(x, y) = x$. Find the directional derivative of π at (0,0) in the direction (3,5). (6+4)

b) i) Prove that a closed subset of a complete metric space is complete.

ii) Show that the integral $\int_0^{\frac{\pi}{2}} \log\sin x\, dx$ convergent and hence evaluate it. (5+5)

c) i) Expand $x + x^2$ in Fourier series on $-\pi < x < \pi$ and deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

ii)Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{-x}$, given $y(0) = y'(0) = 0$ by Laplace transform. (5+5)