Test Examination 2018

Nabadwip Vidyasagar College

Sub: Mathematics (Honours) **Paper: VI** **F.M.-50** **Time: 2 hours**

(1) Answer any **five** questions: 1 x 5 = 5

a) Define the upper Darboux sum of a bounded function f in [a,b]

b) Define compact metric space.

c) State Bonnet's form of the Second Mean Value Theorem.

d) State Young's Theorem.

e) Show that $\frac{9}{10}\Gamma\left(\frac{8}{3}\right) = \Gamma\left(\frac{2}{3}\right)$

f) Examine whether the improper integral $\int_0^\infty \frac{dx}{1+x^2}$ exists.

(2) Answer any **five** questions: 2 x 5 = 10

a) Give an example of a Cauchy sequence in a metric space which does not converge to an element of that space.

b) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n$.

c) Let (X,d) be a metric space. Show that the (X, d_1) is also a metric space, where $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$.

d) Using Weierstrass' M-Test, show that the series $\sum_{n=1}^{\infty} \frac{\cos nx}{n^p}$ (p>1) is uniformly convergent on $(-\infty, \infty)$.

e) Verify implicit function theorem for the function $f(x,y) = x^2 + y^2 - 1$ at (0,1).

f) Given f(x)=0, x is rational

=1, x is irrational

Prove from definition that f is not Riemann integrable on [a,b], a<b.

(3) Answer any **three** questions: 5 x 3 =15

a) Find the complete integral of $px + 3qy = 2(z - x^2p^2)$ by Charpit's Method.

b) Solve (y+z)p + (z+x)q = x + y by Lagrange's method.

c) Using Lagrange's method, find the maximum value of $f(x,y,z) = x^2y^2z^2$ subject to subsidiary condition $x^2 + y^2 + z^2 = c^2$.

d) Expand $f(x,y) = x^2y + 3y - 2$ in power of (x-1) and (y-2).

(4) Answer any **two** questions: 10 x 2 = 20

a) i) Find the Jacobian $\frac{\partial(y_1,y_2)}{\partial(x_1,x_2)}$, where $y_1 = x_1(1 - x_2)$ and $y_2 = x_1x_2$.

ii) Show that the function f, where

$$f(x,y) = \frac{xy}{\sqrt{x^2+y^2}} \text{ if } x^2 + y^2 \neq 0$$
$$= 0, \text{if } x = y = 0$$

is continuous, possesses partial derivatives but is not differentiable at the origin.

iii) Let $\pi: \mathbb{R}^2 \to \mathbb{R}$ be defined by $\pi(x,y) = x$. Find the directional derivative of π at (0,0) in the direction (3,5). (2+5+3)

b) i) Show that Euclidean n-space $\mathbb{R}^n$ is a complete metric space, under a metric specified by you.

ii) Show that the integral $\int_0^{\frac{\pi}{2}} \log\sin x \, dx$ convergent and hence evaluate it. (5+5)

c) i) Expand $x + x^2$ in Fourier series on $-\pi < x < \pi$ and deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

ii)If $f: [a,b] \to \mathbb{R}$ is monotonic, then prove that f is Riemann integrable. (5+5)